EPISTEMOLOGY BELIEFS ABOUT MATHEMATICAL PROBLEM SOLVING AMONG MALAYSIAN STUDENTS

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Abstract: The aim of this study is to investigate student's epistemological beliefs about mathematical problem solving. Specifically, the present study sought answers to the following questions: What is the overall profile of student's epistemological beliefs about mathematical problem solving? Do student's epistemological beliefs about mathematical problem solving differ in terms of gender variable? Do student's epistemological beliefs about mathematical problem solving differ in terms of grade level variable? Do student's epistemological beliefs about mathematical problem solving differ in terms of mathematical ability variable? A total of 120 secondary (form 1, form 3, and form 5) schools students were participated in this study. A Likert Scale compressed 36 items was validated and applied to answer the research questions. Results revealed that: (1) Malaysian students had moderate levels of epistemological beliefs about mathematical problem solving; (2) there were no significant differences among Malaysian students' in their beliefs about problem solving due to gender, academic level, mathematical ability, and race.

Keywords: Epistemological beliefs, Problem solving, Mathematical ability.

INTRODUCTION

Epistemological beliefs refer to beliefs about the nature of knowledge (including its structure and certainty) and knowledge acquisition (including sources and justification of math knowledge) ([2, 3]). Students' epistemological beliefs have become one of the critical components of understanding student learning, deeply influencing and mediating the learning process and the learning outcome ([4, 6, 18, and 47]). Schommer [7] hypothesized a five dimensional structure of epistemological beliefs including three dimensions regarding the nature of knowledge and knowing and two beliefs related to the nature of learning and intelligence.

Epistemological beliefs have been investigated regarding their impact on test comprehension and metacomprehension, problem solving, and conceptual change. Moreover, students' beliefs have been investigated as convictions about knowing and learning in mathematics [8].

Schoenfeld [9] pointed out students' "mathematical worldviews" are key components to the successful completion of problems. Schoenfeld dichotomously defined students' mathematical problem solving beliefs as rational or empirical. In general, Schoenfeld [9, 10] suggested that rational problem solvers exert more control and are more successful than their empirical counterparts. Typical empirical students' beliefs included the assumptions that formal mathematics is not needed during problem solving, mathematics problems are solved quickly or not at all, mathematical discovery is only possible for geniuses, a unique solution exists for all mathematics problems, and an algorithmic, procedural method is available for all mathematics problems.

Furthermore, Shoenfeld [11, 12] noted how the existence of beliefs system drives student's behaviors during problem solving processes; students believe that mathematics problems should be solved in less than twelve minutes, and if not they should not waste time trying, as they would never find the solution.

Liu and Chen [13] showed how the students' beliefs about mathematics consist of three parts: the view of the nature of mathematics knowledge, the view of mathematics learning process and the academic self-concept in the field of mathematics. Li [14] found that four factors, namely, the interest of mathematics learning, the involvement of classroom learning, the views about mathematics, and the quality of classroom discourse, can explain the variance of primary school students' views of mathematics leaning.

A large body of literature indicates that student's epistemological beliefs about learning influence their learning processes and problem solving behaviors [15].

In this study, secondary school students' epistemological beliefs about mathematical problem solving were examined to provide a contribution to the learning process. This study provides opportunity to determine whether there is a gender related difference in students' epistemological beliefs about mathematical problem solving or not. It tries to explore the impact of the academic level (class) on students' beliefs, and to explore the relationship between mathematics achievement and beliefs about problem solving. The results obtained from the current study may be used to enhance students' learning process, learning environment, the teachers' teaching methods, and the counselors' counseling techniques. By knowing the students' epistemological beliefs about mathematical problem solving, the teachers have an idea and may plan instructional activities to develop students' beliefs.

BELIEFS AND MATHEMATICS

Previous researches indicated that students tend to believe that mathematics knowledge is static, the goal of problem solving is to produce the right answer, and mathematics skill is either something you have or do not have [16, 17]. Muis [18] stated that the student's epistemological beliefs about mathematics were concepts in the personal epistemological area, which refers to naive views or opinions about the nature and acquisition of mathematics knowledge. The components of students' epistemological beliefs about mathematics may include the nature of mathematics knowledge, justifications of mathematics knowledge, and sources of mathematics knowledge. Students at all levels hold non availing beliefs. For example, students believe mathematics knowledge is passively handed to them by some authority figure, typically the teacher or textbook author, and they believe those who are capable of doing mathematics were born with a "mathematics gene" (a belief in innate ability). Students who held sophisticated epistemological beliefs about the nature of mathematics knowledge scored higher on a mathematics performance test than did those who held simpler or more naïve beliefs [19]. Lampert [20] pointed out students' common view of mathematics is in terms of certainty and fast and correct answers that become true when accepted by teacher authority.

Beliefs about mathematics were an underrepresented theme in research on how students learn mathematical problem solving [21]. Epistemological beliefs have become a common area of research because they might explain gender differences in mathematics achievement [22]. Stage and Kloosterman [23] found that men and women did not differ significantly in their beliefs, but that beliefs were more strongly related to course grade for women than for men. Mau's [24] study found that many of the students believed that simply memorizing formulas and algorithms was the best way to master course content. In brief, students held "invalid" beliefs about what they should do to master mathematical concepts; those beliefs appeared to be a major reason for difficulty with the course.

McLeod [25] described four types of beliefs about teaching and learning mathematics: (1) mathematics is a discipline consisting primarily of rules and procedures, (2) the beliefs about oneself as a learner of mathematics, self-confidence in learning mathematics, and beliefs about the relationship of gender to interest or mathematical achievement, (3) beliefs about what teachers should do to communicate mathematical ideas, and (4) beliefs about social context including beliefs about what it means to be a student in a mathematics class and beliefs about the way mathematics is seen in school as opposed to non-school settings.

Garofalo [26] indicated different kinds of students' beliefs about mathematical problem solving influenced mathematical achievement, for instance: (1) the level of problem difficulty is due to the size and quantity of numbers, (2) mathematical problems can be solved by performing one or two computational operations, (3) the operation to be performed usually is determined by problem keywords, (4) students' decisions to revise and to check what has been done depends on how much time is available.

Kloosterman and Stage [1] described five beliefs appropriate for the types of mathematics taught in secondary mathematics curricula. The first belief (Difficult Problems) involves confidence in solving time-consuming mathematics problems. The second belief (Steps) concerns whether word problems must be solved conceptually as opposed to employing step-by-step algorithms and is a belief about mathematics itself. The third belief (Word Problems) deals with the discipline of mathematics and the notion that word problems and other non-computational problems are an essential element of mathematics. The fourth belief (Understanding) is a belief about the self as a learner of mathematics through either understanding or simple memorization of mathematical procedures. The fifth belief (Effort) measures the belief about oneself that effort will result in improved mathematical ability and thus long-term success in mathematics.

In the present study, the Kloosterman and Stage model of students' beliefs about mathematical problem solving was validated and used to assess Malaysian students' beliefs about mathematical problem solving.

PROBLEM STATEMENT

Researchers empirically linked sophisticated epistemological beliefs with demographic variables like age and level of education [27, 28] as well as a number of desired learner outcomes including higher mastery test scores, text comprehension [29], conceptual understanding [30], meta-cognitive study strategies [27], academic achievement [31, 32], and gender [33].

Research on students' beliefs was considerably absent in Malaysia in contrast with western countries. Since the social beliefs accumulated in a certain time and region profoundly influence students' epistemological beliefs, as indicated by a cross-cultural research on beliefs [33], there was a need to examine whether the Malaysian students' epistemological beliefs about mathematical problem solving were the same as in other countries. In this study we identified Malaysian high school students' beliefs about mathematical problem solving. We also analyzed possible significant differences in beliefs related to grade (three years of secondary school) and sex, as well as the relationship between beliefs and affect along with achievement in mathematics.

However, evidence available to date strongly suggests that environmental factors such as gender role socialization might be more important. Leder [22] suggested that environmental factors directly influence students' beliefs about the self which are then a key to explaining gender differences in achievement. In short, it is likely that many gender differences in mathematics achievement are related to differences in beliefs.

Researchers stated that beliefs about mathematics should be studied to better understand how students learn problem solving. Students have varying beliefs about mathematics as a subject and about themselves as learners of mathematics. Some of these beliefs make students so interested in learning mathematics, whereas other beliefs hinder their interest and understanding of mathematics [1]. It is also assumed that increasing students' motivation to solve mathematical problems will enable them to become good problem solvers. Also, increase in the likelihood of measuring students' beliefs will allow mathematics teachers to determine the belief of their students and then modify the teaching process to enhance students' beliefs about mathematics.

If Malaysian society is to be more highly industrialized, it may be necessary for Malaysian educators to understand better the nature of epistemological beliefs about mathematical problem solving. Also they may need to find ways to facilitate the development of Malaysian students' learning beliefs. If learning beliefs are to be developed from early on, that is, while students' mind habits or brain functions are more plastic and flexible, then development could be accelerated or facilitated.

The purpose of this study was to examine relationships between students' beliefs about mathematical problem solving and mathematics achievement. This study was designed to extend previous research results that have shown significant relationships between self-beliefs and academic achievement [34]. Furthermore, the present study investigated the effect of gender and academic level on students' beliefs about mathematical problem solving.

Specifically, the present study sought answers to the following questions: (1) What are the overall students' beliefs about mathematical problem solving? (2) Do students' beliefs about mathematical problem solving differ by gender? (3) Do students' beliefs about mathematical problem solving differ by grade level? (4) What is the relationship between mathematical achievement and the following independent variables? (a) Difficult problems (b) Steps (c) Word problems (d) Understanding concepts (e) Efforts (5) From the previously stated independent variables, which ones, individually and in linear combination, predict or best predict students' mathematical achievement?

Table 1: Research samples by gender and class

Variable		Number of students	Percentage %
Gender	Male	272	46
	Female	320	54
	Form one	176	30
Class	Form three	191	32
	Form five	225	38
Total		592	

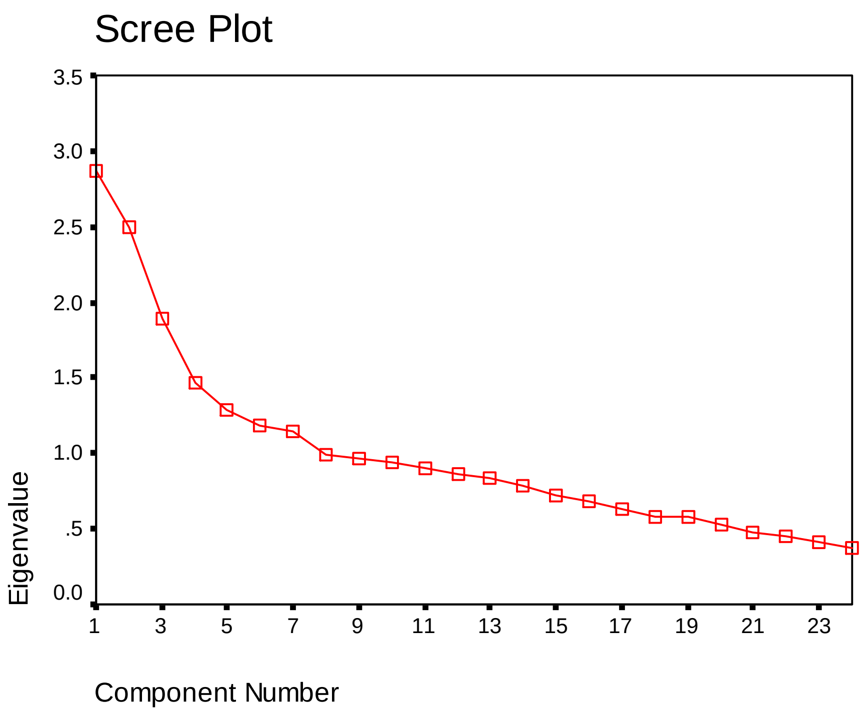

Figure 1: Screen Plot Diagram Showing The Eigen Values Of The Items.

Table 2: Item Communalities and Final Exploratory Factor Analysis Results

Item	Factor number				
	1	**2**	**3**	**4**	**5**
Math problems that take a long time don't bother me	.80				
I feel I can do math problems that take a long time to complete.	.73				
I find I can do a math problem if I just hang in there.	.58				
If I can't do a math problems in a few minutes. I probably can't do it all.	.42				
There are words problems that just can't be solved by following a predetermined sequence of steps.		.75			
Word problems can be solves without remembering formulas.		.72			
Memorizing steps is not that useful for learning to solve word problems.		.69			
Any word problems can be solved if you know the right steps to follow.		.44			
Time used to investigate why a solution to a math problem works is time well spent.			.78		
A person who doesn't understand why an answer to math problems is correct hasn't really solved the problem.			.77		
In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.			.72		
It's not important to understand why a mathematical procedure works as long as it gives a correct answer.			.45		
A person who can't solve word problems really can't do maths.				.69	
Computational skills are of little value if you can't use them to solve word problems.				.65	
Computational skills are useless if you can't apply them to real life situation.				.64	
Learning computational skills is more important than learning to solve word problems.				.79	
Math classes should not emphasize word problems.				.72	
By trying hard, one can become smarter in mathematics.					.71
Working can improve one's ability in mathematics.					.58
I can get smarter in math by trying hard.					.58
Ability in math increases when one studies hard.					.49
Hard work can improve one's ability to do math.					.68

Table 3: Regression Estimate of the First Order CFA of the *IMBS scale*

Item	Estimate	Standard error	*t* value	*p*
Math problems that take a long time don't bother me	1.00**	-	-	-
I feel I can do math problems that take a long time to complete.	1.02	.09	12.05	.00
I find I can do a math problems if I just hang in there.	1.07	.08	13.51	.00
If I can't do a math problem in minutes, I probably can't do it at all.	.99	.84	11.70	.00
There are word problems that just can't be solved by following a predetermined sequence of steps.	1.00**	-	-	-
Word problems can be solved without remembering formulas.	.98	.08	10.79	.00
Memorizing steps is not that useful for learning to solve word problems.	1.08	.08	12.62	.00
Any word problems can be solved if you know the right steps.	1.31	.24	4.59	.00
Time used to investigate why a solution works is time well spent.	1.00**	-	-	-
A person who doesn't understand why an answer to math problems is correct hasn't really solved the problem.	.87	.19	1.88	.05
In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	.63	.31	4.74	.00
It's not important to understand why a mathematical procedure works as long as it gives a correct answer.	1.31	.26	5.17	.00
A person who can't solve word problems really can't do maths.	1.00**	-	-	-
Computational skills are of little value if you can't use them to solve word problems.	.87	.32	4.28	.00
Computational skills are useless if you can't apply them to real life.	.63	.22	4.21	.00
Learning computational skills is more important than learning to solve word problems.	1.33	.41	4.58	.00
Math classes should not emphasize word problems.	1.26	.31	4.29	.00
By trying hard, one can become smarter in mathematics.	1.00**	-	-	-
Working can improve one's ability in mathematics.	.64	.26	3.63	.00
I can get smarter in math by trying hard.	.54	.28	2.46	.01
Ability in math increases when one studies hard.	1.12	.43	3.57	.00
Hard work can improve one's ability to do math.	1.32	.37	3.41	

Table 4: The Correlation Matrix between Independent Variables

Variable	Difficult problems	Steps	Understanding	Word problems	Effort
Steps	.143**				
Understanding	.082*	.081*			
Word problems	.185**	.200**	.217**		
Effort E	.313**	.103*	.144**	.005	
Achievement	.617**	.479**	.431**	.518**	.668**

** Correlation is significant at the .01 level (2-tailed).
* Correlation is significant at the .05 level (2-tailed).

Table 5: Descriptive Statistics for Each Subscale by Gender and Class.

		Difficult problems	Steps	Understanding	Word problems	Effort	Beliefs
Male	Mean	3.56	3.54	3.65	3.49	4.31	3.73
	SD	2.77	2.26	2.15	2.66	3.94	7.67
Female	Mean	3.68	3.68	3.66	3.52	4.37	3.79
	SD	2.62	2.33	1.95	2.89	3.97	8.02
Form 1	Mean	3.65	3.61	3.68	3.48	4.46	3.79
	SD	2.99	2.27	1.97	2.73	3.69	8.00
Form 3	Mean	3.63	3.66	3.62	3.71	4.27	3.78
	SD	2.58	2.16	1.96	3.12	4.04	7.68
Form 5	Mean	3.69	3.57	3.66	3.45	4.30	3.73
	SD	2.55	2.46	2.16	2.46	4.07	8.02
Total	Mean	3.62	3.61	3.65	3.51	4.34	3.76
	SD	2.70	2.31	2.04	2.78	3.97	7.92

METHODOLOGY

Sampling

The participants for this study were 592 secondary school students from different public schools in Kuala Lumpur, Malaysia. Forty six percent (46%, $N = 272$) of the participants were females while fifty four percent (54%. $n = 320$) were males. Thirty percent (30.2%; $N = 176$) of the participants were from Form one class, thirty two percent (32%, $N= 191$) were from Form three class, and thirty eight percent (38%, $n = 225$) were from Form five class. Table 1 shows the research samples distributed by gender and class.

Research Design

The research design for the study used correlational and multivariate analyses followed by univariate analysis to understand the relationships among important variables. The use of quantitative correlational and multivariate analyses was considered appropriate for determining whether the observed variables covaried, which would establish the direction, magnitude, and form of the observed relationships. Furthermore, this study also attempted to determine whether significant relationships existed between beliefs and student mathematics achievement. Mathematics achievement was measured by the students' grades in mathematics from school documents. To explore the effects of the independent variables on more than one dependent variable, a multivariate analysis was conducted. Multivariate analysis was appropriate because it allowed for the assessment of the effects of a number of different independent variables on single or multiple dependent variables using one test.

Sex, the first independent variable, was dichotomized into male and female, whereas class, the second independent variable, was divided into three levels: Form one, Form two, and Form three. The dependent variable was beliefs about mathematical problem solving.

The present study extracted the relationship between beliefs about mathematical problem solving and academic achievement using Pearson's correlation coefficient, and the significance of the correlation coefficient was tested. Furthermore, multiple regression was used to find out the best predictors of mathematics achievement.

Instrument

The Indiana Mathematics Belief Scale (IMBS) developed by Kloosterman and Stage [1] was adopted and validated to assess students' beliefs towards mathematical problem solving. The scale consists of thirty Likert- type statements representing beliefs towards problem solving (Difficult problems, Steps, Understanding, Word problems, and Efforts). Participants responded to the IMBS using a five-point scale of strongly disagree (1), disagree (2), neutral (3), agree (4), and strongly agree (5). The negative items were reverse coded. The IMBS was validated using factor analysis and internal consistency.

Exploratory Factor Analysis (EFA)

Exploratory Factor Analysis is used to uncover the underlying structure of a relatively large set of variables. Principal Component Analysis was employed aiming at empirically revealing and demonstrating the hypothesized, underlying structure of the preliminary model of the questionnaire. Before conducting factor analysis, the results of the *KMO* measure of sampling adequacy and Bartlett's test of sphericity were examined to determine the appropriateness of factor analysis. Bartlett's test was significant (*BTS* value = 2487.21, $p < .001$), showing that the correlation matrix was significantly different from an identity matrix. Similarly, the *KMO* Measure of Sampling Adequacy of .79 was substantial. Both revealed that it was appropriate to perform a factor analysis. A varimax rotation was then undertaken to assist in the interpretation of the factors.

From the scree plot (Fig. 1) and the Kaiser-Guttman rule, factor analysis of results on the 30 items indicated that five of eight factors were interpretable. The rotated principal factor loading matrix for the questionnaire items was shown in Table 1. Four items were assigned to factor one which reflected belief one (I can solve consuming mathematics problems). Four items were assigned to factor two which reflected belief two (There are word problems that cannot be solved by simple, step- by- step procedures). Four items were assigned to factor three which reflected belief three (Understanding concepts is important in mathematics). Five items were assigned to factor four which reflected the belief four (Word problems are important in mathematics). Five items were assigned to factor five which reflected belief five (Efforts can increase mathematical ability).

However, with respect to the theoretical framework from which the items were created, after a careful investigation of the item content, some factors had less than three items with substantial loadings and had split loadings on another factor as well. For these reasons, the other three factors solution did not appear to be the best representation of the structure of the questionnaire. Examination of the item loadings, of items with substantial loadings on more than one factor, and of the actual wording of items that ended up being grouped together led to the determination of the five-factor solution as the best (see Table 2). The overall percentage of variance extracted (53.98%) supported the assertion that the five factors were deemed sufficient and conceptually valid in their correspondence to the existing theory. All items had pattern coefficients higher than .30. Further, reliability coefficients for each factor all exceeded the threshold of .80 for acceptance [35].

Confirmatory Factor Analysis (CFA)

Confirmatory Factor Analysis seeks to determine if the number of factors and the loadings of measured (indicator) variables on them conform to what is expected on the basis of pre-established theory. A CFA was conducted to test the fit between the five-factor model and the data. Analysis was performed using *AMOS 7.0* [48] using maximum likelihood (ML) as the estimation procedure. In terms of the sample size required to use the ML estimator appropriately, Ding, Velicer, and Harlow [36] recommended that the minimum sample size to use MLE appropriately should be between 100 to 150 participants. Following the recommendations of Boomsma [37] and McDonald and Ho [38], several fit criteria were applied. Multiple goodness-of-fit tests were used to evaluate the fit between the hypothesized model and the data to determine if the model being tested should be accepted or rejected. These were chi-square test, the root mean square error of approximation (RMSEA), which is a measure of the discrepancy per degree of freedom between the model and the covariance-matrix in the population, the standardized root mean square residual (SRMR), which is the average difference between observed and reproduced correlations, the non-normed fit index (NNFI), indicating the proportional improvement of the fit of the model relative to the independence model, and the Comparative Fit Index (CFI), which assesses the relative improvement in fit of the researcher's model compared with the baseline model.

The model fit was considered acceptable when both $SRMR < 0.08$ and $RMSEA < 0.06$ [39]. Both the NNFI and CFI should be at least .90. Results of the CFA indicated a good model fit (chi-squared = 201.69, df= 188, $p < .132$; $NNFI = .80$; $CFI = .97$; $RMSEA = .02$; $SRMR = .06$). Table 3 shows the regression estimates and the t values of the items and their respective scales.

Internal Consistency

Internal consistency of the five-factor model of the questionnaire was checked by calculating alpha reliability coefficients using SPSS 11.5. The overall alpha coefficient of .86 was good. There were no items whose elimination would have improved the coefficient substantially. The individual alpha coefficients for different scales ranged from .80–.85, indicating satisfactory reliability. Furthermore, examining item-total correlations indicated that all items in each dimension (subscales) contributed to the consistency of scores with item-total correlations higher than .69.

In the present study, the correlation coefficients were interpreted by employing Davis [40] descriptors (negligible = .00 to .09; low =.10 to .29; moderate = .30 to .49; substantial = .50 to .69; very strong = .70 to 1.00). The relationship among the subscales is shown in Table 4. Accordingly, all correlation coefficients range from .01 to .313 (negligible or low). This suggests that the five subscales were fairly independent to be used as independent variables. This allows us to examine mathematics beliefs of students by each subscale.

RESULTS

Students' Beliefs Profile

Overall profile of the participants' beliefs about mathematical problem solving were measured in terms of the Difficult problems, Steps, Understanding, Word problem, Effort, and Overall beliefs. The mean scores and standard deviations were used to explain the students' Beliefs profile. According to Birisci, Metin, and Karakas [41], ranges of agreement with the attributions on the survey was determined by using the (n-1)/n formula (n is the number of scale levels) and after calculation the interval width of the range between 1 through 5 was calculated as 0.8. As such, the interval width of 1-1.80 showed very low level, the 1.81-2.60 interval showed low level, the 2.61-3.40 interval showed medium level, the 3.41-4.20 interval showed high level and the 4.21-5.00 interval showed very high level of agreement with the statement on the survey**.** As can be seen in Table 5, the results of the descriptive statistics indicated that participants' beliefs about problem solving were given as mean scores ranging from 3.51 to 4.34 on a five point scale. *Effort* dimension had the highest mean value (*M*ean = 4.34, *SD* = 3.97), followed by *Overall beliefs* (*M*ean = 3.76, *SD* = 7.92). Furthermore, the means suggest that students have high levels in four dimensions: Difficult problems (*M*ean = 3.32, *SD* = 2.70), Steps (*M*ean = 3.31, *SD* = 2.31), Understanding (*M*ean = 3.35, *SD* = 2.04), and Word problems (*M*ean = 3.21, *SD* = 2.78). Students have a very high level in efforts belief (*M*ean = 4.34, *SD* = 3.97). At the global level, the overall student's beliefs are well above the mid-point of the scale (3.00) and this indicated that students held positive (sophisticated) beliefs about mathematical problem solving.

Table 6: Results of Multivariate Tests

Effect	Wilks lambda	*F*	Hypothesis df	Error df	*p*-value
Gender	.980	2.387	5.000	582.000	.037
Class	.967	1.982	10.000	1164.000	.032

Table 7: Tests of Between-Subjects Effects (MANOVA Results by Gender and Class)

Source	Dependent variable	Type III Sum of Squares	*Df*	Mean square	*F*-value	*p*-value
Gender	Difficult problems	34.377	1	34.377	4.744	.030
	Steps	44.960	1	44.960	8.466	.004
	Understanding	.019	1	.019	.004	.947
	Word problems	1.973	1	1.973	.258	.612
	Efforts	13.822	1	13.822	.880	.349
Class	Difficult problems	5.657	2	2.828	.390	.677
	Steps	16.947	2	8.474	1.596	.204
	Understanding	4.192	2	2.096	.503	.605
	Word problems	80.550	2	40.275	5.263	.005
	Efforts	92.767	2	46.384	2.954	.053

Table 8: Summary Results of Two-way Analysis of Variance

Source	Type III Sum of Squares	Df	Mean Square	*F*	Sig.
Gender	308.164	1	308.164	4.983	.026
Class	250.203	2	125.102	2.023	.133
Gender* Class	295.326	2	147.663	2.388	.093
Error	36238.735	586	61.841		
Total	3472300.000	592			

Table 9: Standard Regression Model Summary

Model	*R*	*R* Square	Adjusted *R* Square	Std. Error of the Estimate
1	.814	.662	.659	5.28073

Table 10: ANOVA: Regression Significance

	Sum of Squares	Df	Mean Square	*F*	Sig.
Regression	31999.221	5	6399.844	229.499	.000
Residual	16341.291	586	27.886		
Total	48340.512	591			

Table 11: Regression Coefficients of Standard Regression Model

	Unstandardized Coefficients		Standardized Coefficients		
Variable	**B**	**Std Error**	**Beta**	***t*- value**	**Sig**
Constant	3.974	2.206	-	1.802	.070
Difficult problems	1.002	.087	.299	11.571	.000
Steps	1.067	.097	.273	11.047	.000
Understanding	.895	.110	.202	8.129	.000
Word problems	.911	.083	.281	11.045	.000
Efforts	.842	.058	.370	14.417	.000

Table 12: Correlations Coefficients and Beta Values

	Correlations			
Variable	**Zero-order**	**Partial**	**Part**	**Part squared**
Difficult problems	.522	.431	.278	.077
Steps	.427	.415	.265	.070
Understanding	.363	.318	.195	.038
Word problems	.437	.415	.265	.070
Effort s	.522	.512	.346	.119

MANOVA analysis

Assumptions were checked before conducting Multivariate analysis (two-way MANOVA). MANOVA has seven assumptions: sample size, independence of observations, normality, outliers, linearity, multicollinearity and singularity, and homogeneity of variance-covariance matrices. No violations were found on multivariate normality and equality of variance.

A multivariate analysis was conducted to investigate the effects of gender, and class, on participants beliefs about mathematical problem solving. In order to evaluate multivariate significance, Wilks Lambda statistic was used. MANOVA results regarding the gender, and class are presented in Table 6. The results indicated statistically significant effect of gender on the combined dependent variables ($F(5, 587) = 2.387$, Wilks lambda = .980, $p = .037$). Also, there is statistically significant effect of class on the combined dependent variables ($F(10, 577) = 1.982$, Wilks lambda = .967, $p = .032$).

In order to investigate on which dependent variables participants in different group of gender and class differed in their beliefs about mathematical problem solving, multivariate analyses of variance between groups was conducted. Table 7 shows the summary results of MANOVA analysis. As seen in Table 7, females scored significantly higher than males in Difficult problems $F(1, 590)= 4.744$, $p < 0.05$; and Steps $F(1, 590)= 8.466$, $p < 0.05$), whereas males and females are simillar in Understanding $F(1, 590)= .004$, $p > .05$; Word problems $F(1, 590) = .258$, $p > .05$; and Efforts $F(1, 590) = .880$, $p > .05$. The three classes are similar in Difficult problems $F(2, 589) = .390$, $p > 0.05$; Steps $F(2, 589) = 1.596$, $p > .05$; Understanding $F(2, 589) = .503$, $p > .05$, and efforts $F(2, 589) = 2.954$, $p > .05$**,** whereas, the three classes differ in Word problems $F(2, 589) = 5.263$, $p < .05$. Resuts from post hoc analysis revealed that form three students scored significantly higher than form five students in word problems beliefs.

To explore the effect of the class and gender variables on epistemological beliefs abuot problem solving, two way analysis of variance was used. Table 8 shows that females scored significantly higer than males in their beliefs about problem solving $F(1, 590) = 4.983$, $p < .05$. Whreas, students in different classes are similer in their beliefs about problem solving $F(3, 151) = 2.388$, $p > .05$).

Correlation Analysis

The Pearson Product Moment correlation coefficients were used to represent the relationship between mathematical achievement and the independent variables: (a) Difficult problems, (b) Steps, (c) Understanding, (d) Word problem, and (e) Efforts. There were substantial positive relationships between mathematical achievement and: Difficult problems ($r = .617$, $p < .01$); Word problems ($r = .518$, $p < .01$); and Efforts ($r = .668$, $p < .01$); however, there were moderate positive relationships between mathematics achievement and Steps ($r = .479$, $p < .01$); and Understanding ($r = .431$, $p < .01$).

Regression Analysis

Before conducting multiple regression analysis, some assumptions have been checked, they include lack of multicollinearity, normality, linearity, homoscedasticity, influential points and outliers, and independence of participants' scores [42]. No violation for conducting multiple regressions was found.

Hence multiple regression analysis was conducted to identify the best predictors of the dependent variable and to show the proportion of variance in the dependent variable (mathematics achievement) explained by the independent variables (Difficult problems, Steps, Understanding, Word problem, and Effort). A direct method entry was used for the multiple linear regression analyses. The standard multiple regression with a direct method entry was used to measure the relationships among variables.

The summary of the multiple regression results are presented in Table 9 and Table 10. R^2 is called multiple correlation" ([42], p. 231), R is a measure of the association between the dependent variable and independent variables [42]. R Square (R^2) represents the proportion of the dependent variable's variance, which is accounted by the linear combination of the independent variables [42]. Adjusted R Square (R^2) is the population R^2 that can be used to generalize the findings from the sample [42]. The results indicated that 66% of the variance in mathematics achievement was explained by the independent variables. The test statistic was significant at the .01 level of significance ($F(5, 586) = 229.499$; $p = .000$).

The standardized regression coefficients (Beta), give an indication of the contribution of each independent variable in predicting the dependent variable [43] (Table 11). The Sig (p) for each independent variable represent a measure of the significance of this variable in predicting the dependent variable.

For the first independent variable (Difficult problems), the test was statistically significant ($t = 11.571$, *Beta* = .299; $p = .00$). This suggested students' beliefs about Difficult problems were significantly predictor of mathematics achievement.

For the second independent variable (Steps), the test was statistically significant (t = 11.047, *Beta* = .273; p = .00). This suggested students' beliefs about Steps were significantly predictor of mathematics achievement

For third independent variable (Understanding), the test was statistically significant (t = 8.129, *Beta* = .202; p = .00). This suggested students beliefs about Understanding were significantly predictor of mathematics achievement.

For the fourth independent variable (Word problems), the test was statistically significant (t =11.045, *Beta* =.281; p = .00). This suggested students' beliefs about Word problems were significantly predictor of mathematics achievement.

For the fifth independent variable (Efforts), the test was statistically significant (t = 14.417, *Beta* = .370; p = .00). This suggested students' beliefs about Efforts were significant predictors of mathematics achievement.

To determine the best predictors among the independent variables in predicting the dependent variables, standardized regression coefficients (Beta), partial correlation coefficients, and part correlation coefficients were used. Table 12 shows that Efforts has the greatest value of Beta, partial correlation coefficient, and part correlation coefficient. Efforts variable was the best predictor of dependent variable that had the most significant effect in predicting mathematics achievement. This predictor accounted to 11.9% of the total variance of students' beliefs after controlling for the other four variables in this study. The second most important predictor in predicting mathematics achievement was Difficult problems variable that accounted for 7.7% of the total variance of mathematics achievement after controlling for the other four variables in this study. Steps and Word problems accounted for 7% of the total variance of mathematics achievement. Understanding meanwhile accounted for 3.8% of the total variance of mathematics achievement.

DISCUSSION

Beliefs about mathematical problem solving were measured in terms of difficult problems, Steps, Understanding, Word problems, and Efforts. At the global level, the overall beliefs about mathematical problem solving are above the mid-point of the scale (3.00) and this indicated that students held positive beliefs about mathematical problem solving.

For the Difficult problems belief, students believe they are confident in solving time-consuming mathematics problems. Students have motivation to try to solve mathematical problems regardless of the time. So it is important to consider students' beliefs about their ability to solve problems which take more than a minute or two to complete [1]. For the Steps belief, students believe that word problems must be solved conceptually not only by applying rules or algorithms. For the Word problems belief, students believed that word problems and other non-computational problems (nonroutine problems) are an essential element of mathematics.

For the Understanding belief, students considered themselves as learners of mathematics through either understanding or simple memorization of mathematical procedures. Students who do not feel the importance of understanding why a particular procedure works and rely on memorizing procedures in solving mathematical problems will be less motivated to try to learn mathematics, whereas students who take the time to understand why a particular procedure works will know they have an ability for learning and understaning mathematics; thus they will be motivated to try and learn mathematics.

For the Effort belief, students believed that efforts will result in improved mathematical ability and thus long-term success in mathematics. According to our experience in teaching mathematics, some students believe that they lack ability to learn mathematics, whereas others believe that they can learn mathematics and improve mathematical ability with sufficient effort. As such, students who believe that studying will not improve their mathematical ability will be less motivated to try to learn mathematics. In contrast, students who believe that studying increases their mathematical ability will have a high level of motivation to excel in mathematics.

Students' sophisticated beliefs about problem solving may be due to the mathematics curriculum and teaching and evaluation processes. For instance, mathematics textbooks pay more attention to relating mathematical problem solving to real life situations with which the students are familiar. The contents are introduced through the description of real life situations and a number of open-ended questions are included in the textbooks. This enables students to understand more clearly that mathematical problem solving skills are dynamic and closely related to real life.

Early findings from the mathematical beliefs research documented common perceptions of mathematics held by students. The findings were generally consistent and indicated that students at all levels of instruction viewed mathematics as the memorization of a variety of algorithms [44]. A related belief that emerged in these studies, that occurs perhaps as a consequence of the belief that mathematics is collection of algorithms, is the idea that problems should be solved quickly. Students tend to believe that if they cannot solve a problem in 5 to 10 minutes, the problem is beyond their range of knowledge or ability [12, 45]. Researchers consistently found it difficult for students to conceptualize mathematics apart from computation [45, 46]. Students who held sophisticated epistemological beliefs about the nature of mathematics knowledge scored higher on a mathematics performance test than did those who held simpler or more naïve beliefs [19].

The findings revealed that there are significant differences between male students and female students regarding their beliefs about Difficult problems and Steps dimensions; the scores are above the average, which indicates that on one hand male students' and female students' beliefs about mathematical problem solving are in substantial agreement, and on the other hand their beliefs need further development. Concretely, scores of female students' beliefs on Difficult problems and Steps dimensions are slightly higher than those of male students', but on the dimensions of Understanding, Word problems, and Efforts there is no statistically significant difference. Accordingly, the existence of the bias that female students were not born to learn mathematics is invalid.

MANOVA results revealed significant difference among students' beliefs about Word problem due to class level. Results from post hoc analysis indicated that form three students have more sophisticated beliefs than form five students about word problems. The change of course content and evaluation of mathematics learning from form three classes to form five classes may account for this tendency. In terms of curriculum, form three mathematics textbooks pay more attention to relating mathematical knowledge to real life situations familiar to the students. Therefore, they got higher scores in the dimension of the beliefs of word problems. In terms of evaluation of mathematics learning, form three students are under much less academic pressure than form five students.

The findings from multiple regression revealed that the five scales significantly predict mathematical achievement to a different extent. The strongest predictor was belief regarding the role of effort in increasing mathematical ability. One possible interpretation may be that students were convinced that hard work is effective and apply it. And Malaysian students are motivated enough to do their best. Students' beliefs about mathematical problem solving may be interconnected very strongly with their mathematics course. It was like an invisible hand, deeply hiding behind an individual's behavioral expression, cognitive process and emotional experience, but deeply affecting the learning process and performance [5, 18]. It appears that a person's epistemological orientation influences engagement, motivation, and academic performance

All dimensions of beliefs about mathematical problem solving are significantly correlated with mathematics achievement. Similar to Western research results, however, epistemological beliefs predict mathematics achievement very well. Mason [8] indicated that the strongest predictor was belief regarding perceived ability to solve mathematics problems. The more students believe in their ability, the better their mathematics grades. Muis [18] pointed out that beliefs about mathematical problem solving could strongly predict mathematics achievement. The more students believed in the conviction that mathematics knowledge was isolated and eternal, the worse their mathematics achievement. Achievement of students, who believed in the relativity and constructed nature of mathematics, was better than that of those who believed in dualism and accepted nature. Students became more and more convinced that not all problems can be solved by applying routine procedures; this led to the progressive increase in their beliefs that it is possible to solve difficult problems.

Investigating beliefs is important since they are behind students' attitudes toward mathematical activities and achievement. In particular, students with low achievement may be unaware of their implicit, maladaptive representations about mathematical problems, and be less able to modify them, so these beliefs negatively affected their learning and achievement. Also, adequate educational interventions should be developed and implemented in mathematics instruction to gradually change naïve representations about the nature and acquisition of knowledge in mathematics, Finally, in order to modify students' naïve beliefs about mathematical problem solving, mathematics teachers should emphasize students' understanding of concepts, effort that increase mathematical ability, and control over learning process and problem solving skills [8].

Students' epistemological beliefs about mathematical problem solving is an important component of their learning experience since they affect the students' involvement in the mathematics learning activities as well as their mathematics performance. Being aware of how the students view mathematical problem solving, the teachers can provide more appropriate and effective instruction for the students in mathematic learning. Teachers may be able to help the students with poor performance become more interested in problem solving by cultivating their mathematical beliefs.

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