INABILITY TO HANDLE SIMPLE MATHEMATICS AMONG LOWER SECONDARY SCHOOL STUDENTS A CAUSE FOR WORRY

Noraini Noordin [a], Rohana Dollah [b], Rudzah Lebai Talib [c]
[a, b, c] Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara (PERLIS), Malaysia.
[a] Corresponding author: noraininoordin@perlis.uitm.edu.my

 ISSN 1923-6654 (print)
ISSN 1923-6662 (online). Available at http://www.ssrn.com/link/OIDA-Intl-Journal-Sustainable-Dev.html

Abstract: Development of number sense in students at an early age has been the concern of most educators worldwide including Malaysia. The school system produced students who performed on examinations, but are lost when given a problem that is not from a textbook. There is cause to worry because students may not understand enough of numbers to proceed to higher mathematics. This has already been proven by research works in this area. Development of number sense among students can help to identify the proficient learners. Students who have developed number sense display some defined characteristics, such as a) able to focus more on strategies than a right answer, b) able to think instead of operating with mathematical rules, and c) able to work at finding his own solution than waiting to be provided with one by the teacher. Studies on number sense carried out from time to time help teachers and educators understand the minute details of problems students go through coping with numbers. This paper presents some of the findings from a study carried out at six schools situated in the northern zone of Malaysia in the effort to discover whether the acquisition of such proficiency existed among the lower secondary school students who have undergone six years of Mathematics at primary schools with respect to some topics under Numbers, Measures, and Shape and Space.

Keywords: Ability Difference, Estimation, Logit, Mental Computation, Number Sense

INTRODUCTION

A school curriculum describes the set of courses offered at the school and their contents. The Primary School Curriculum for Mathematics (*Mathematics Year 1*, 2002; *Mathematics Year 2*, 2003; *Mathematics Year 3*, 2003; *Mathematics Year 4*, 2006; *Mathematics Year 5*, 2006; *Mathematics Year 6*, 2006) requires teachers to present the learning objectives "in a developmental sequence to enable pupils to grasp concepts and master skills essential to a basic understanding of mathematics." It also prescribes the measurable "knowledge, skills or mathematical processes and values" that should be inculcated at the appropriate levels of the learning process. Under points to note, teachers are encouraged to pay attention to significant aspects of mathematical concepts and skills to ensure effective learning. However, it has been noted that the present system has been producing students who were "good at doing examinations" but will be lost when given a different problem from what they found in textbooks (Abu Hasan, 2007). There is a growing global concern among educators as to whether primary school students understand numbers or are just applying procedural algorithms to a mathematical problem (Ghazali, Idros, & McIntosh, 2004). Mathematics deal with problems and how to solve them; the development of these soft-skills in students coupled with the flexibility in the curriculum will promote paradigm shift in mathematics education (Abu Hasan, 2007).

Table 1: Focus of Items in Computation Test

Items	Focus
1-6	Addition and subtraction operations on numbers
7-11	Multiplication and division operations on numbers
12	Addition of two mixed numbers
13	Multiplication of two mixed numbers
14	Mixed operations on decimal numbers
15	Use and apply knowledge of money in real life situation.
16	Finding volume of three-dimensional shapes.
17	Application of fractional computation to problems involving volume of liquid.

National Council of Teachers of Mathematics (NCTM, 2000) states that if a teacher focuses on the application of algorithm, the student will forever be dependent on the specific method that has been taught, thus the teacher should pay more emphasis on the overall thought processes that lead to the correct answer even if the process produces minor mistakes. Some less proficient students will be trapped in the procedural approach but some higher ability students will develop greater flexibility by seeing the whole process and compressing them into concepts that are thinkable(Tall, 2008), thus students who understand the concepts underlying the methods they use will be able to move forward into other spectrums of knowledge(Tall, 2008).

The development of number sense is important in mathematics education. Development of number sense can be observed in students. When students develop number sense, they demonstrate this by showing improvement in their understanding of numbers by being better at representing numbers, associating numbers with number systems, associating between operations of numbers, can compute more fluently and estimate reasonably (NCTM, 2000). Studies have defined characteristics to describe a student who has developed number sense. Among them are a) students are more focused on strategies than on a right answer, b) students prefer to set their minds thinking instead of operating with rules, and c) students are more concerned on generating his own solution rather than on solutions provided by the teachers and the development of these abilities in students should begin at an early age (Ghazali, et al., 2004). There is a need to discover whether the acquisition of such proficiency existed among the lower secondary school students who have undergone six years of Mathematics at primary schools. This paper describes the findings from a study done on lower secondary school students to assess how they performed on a Computation Test, consisting of 17 items from three areas in the Integrated Curriculum for Primary Schools in Malaysia, namely Numbers, Measures, and Shape and Space. Table 1 displays the Focus for Items addressed in this study (*Mathematics Year 1*, 2002; *Mathematics Year 2*, 2003; *Mathematics Year 3*, 2003; *Mathematics Year 4*, 2006; *Mathematics Year 5*, 2006; *Mathematics Year 6*, 2006).

Tsao (2004) explained that development of number sense in a student is affected by his estimation ability, his computation ability, his mental computation and other affective issues. Mental computation can be characterized by its' ability to produce exact answers and its' independency of the need for external aids like pencil and paper (Reys, 1984). Mental computation is not an inherent trait, experiences and practice will help develop strategies that are more sophisticated than traditional written methods (A. J. McIntosh, 2002).

Reys (1984) defined estimation as a) a quick process, b) is generally performed mentally without using pencil and paper, c) does not produce exact answers but provides the basis for making decisions, d) often display individual methods, and e) produces a range of estimates to answers. When mental computation is used in an estimation procedure, mental processes involving selected simple numbers take place to produce approximate answers (Segovia & Castro, 2009), thus, tying a knot between estimation and mental computation. Estimation helps pave the way for the development of clarity in thinking and discussion as well as ability to be consistent in procedural applications. Thus, students would be able to regard Mathematics as a distinct way of thinking and give it a place of importance in our technological society (Bana & Dolma, 2004).

Segovia and Castro (2009) defined estimation as either computational estimation or measurement estimation. Computational estimation referred to arithmetic operations and how one judge the meaning of its results while measurement estimation referred to judgment made on results found after taking measurements. It is important to point out that developments with regards to number sense experienced by primary school students will affect their understanding of mathematics later on in their lives.

Mental computation (or calculation) is emphasized in the primary school curriculum from Years 1 to 6 for all operations (*Mathematics Year 1*, 2002; *Mathematics Year 2*, 2003; *Mathematics Year 3*, 2003; *Mathematics Year 4*, 2006; *Mathematics Year 5*, 2006; *Mathematics Year 6*, 2006). It is also emphasized in the curriculum specifications for lower secondary Mathematics for Form One in topics such as whole numbers, decimals, integers, basic measurements, perimeter and area and solid geometry (*Mathematics Form 1*, 2002). The responses of students were analyzed to understand the pattern of thinking processes that took place and how capable were students at computing, estimating and mental computation. The respondents were only given approximately three minutes for each item as a means to encourage application of mental computation.

METHODOLOGY

Sample

The study was done to assess the computational and estimation abilities of lower secondary school students in Malaysia. The samples were 298 randomly selected Form One students from 6 schools (indexed as SMKBBS, SMKAK, SMKAA, SBPI, SMKD and SMKJ) in the northern zone of Malaysia. These schools included two Religious Schools (SMKAK and SMKAA), one Residential school (SBPI), one "Cluster" school (SMKD) and two Day schools (SMKBBS and SMKJ).

Residential schools are for students with a minimum qualification of a 4A 1B in the Primary School Assessment examination (UPSR) ("Sejarah Sekolah Berasrama Penuh," n.d.). Cluster schools are for students who achieve good results in their UPSR but were not accepted into boarding schools or who did not want to go to boarding schools. The eligibility for entrance into these schools is very high. They usually require 5A in the UPSR from students of Sekolah Rendah Kebangsaan (SRK) and 7A in the UPSR for students who attend Sekolah Rendah Jenis Kebangsaan (SRJK). However, the entrance qualification for Religious Schools is a bit lower than that of a Cluster school., that is at least a 3A and 2B grade in the UPSR and a pass in the qualification test.

Data collection

The respondents in this study sat for two tests: a Computation Test and an Estimation Test. At the end of this test, a few respondents were randomly picked out to sit for a Probing Interview. There were 20 items on the Estimation Test and 17 items on the Computation Test. There were 14 similar items on both tests. All these items in both tests were built based on topics in the Mathematics curriculum for Primary Schools covering two areas, Numbers and Measures(*Mathematics Year 1*, 2002; *Mathematics Year 2*, 2003; *Mathematics Year 3*, 2003; *Mathematics Year 4*, 2006; *Mathematics Year 5*, 2006; *Mathematics Year 6*, 2006).

Data Analysis

The Rasch Measurement Model was used to analyze the responses. This model uses the interaction between respondents and items to determine the probability of success of each person on each item of a test, thus providing a description in the form of a map of the locations of persons and items (Bond & Fox, 2007). In this study, the location of an item on the map provides a measure of the degree of difficulty respondents were facing when asked to respond to the item.

The items were coded using the "0 = wrong" and "1 = right" format. Under Rasch analysis, this coding sets the basis for differentiating a correct and incorrect answer and the correct answer is better and superior than the incorrect answer. It also implies that the respondent with the correct answer has demonstrated more ability than those who do not. Non-responses were treated and coded as incorrect answers; assumption made was that questions were unanswered because respondents did not have the necessary ability to complete the task(Bond & Fox, 2007).

Item reliabilities for all schools measured higher than 0.75. Item reliability is not dependent on the length of a test. The value for item reliabilities in this study can imply a few things. The range of difficulty between items of these tests may be wide and the samples may be large but consistency can be expected of these inferences. It can also mean that the item ordering has a very high probability of being replicated if these same items were given to a different group of respondents (Bond & Fox, 2007).

On the contrary, only three of the schools measured person reliabilities higher than 0.5 but less than 0.75. There are always cases of misbehavior by the respondents in a test. In this case, since reliability is the reproducibility in ascending order of a set of measures based on the ratio of true variance of the measures to observed variance of the measures, the low person reliability may have been due to the existence of too few items in the test or the test is not long item or there are not many categories per item ("Winsteps Help for Rasch Analysis," 2011).

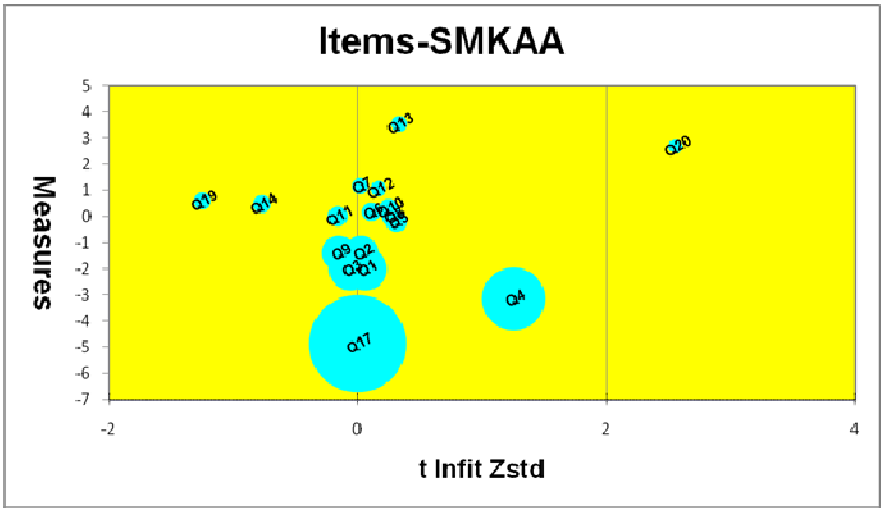


Figure 1: Bubble Chart for Responses from Students of SMKAA

Table 2: Measure order for all items in the test for SMKAA

TABLE 13.1 DATA - COMPUTATION - 2010 - SMKAA - PR ZOU313WS.TXT Jul 20 16:53 2011
INPUT: 57 Persons 17 Items MEASURED: 57 Persons 17 Items 2 CATS 1.0.0

Person: REAL SEP.: .89 REL.: .44 ... Item: REAL SEP.: 2.16 REL.: .82
Item STATISTICS: MEASURE ORDER

ENTRY NUMBER	RAW SCORE	COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEA CORR.	EXACT MATCH OBS%	EXACT MATCH EXP%	Item
13	14	57	3.55	.33	1.05	.3	1.23	.8	.27	77.2	77.6	Q13
17	23	57	2.68	.30	1.32	2.6	1.48	1.6	.12	54.4	69.2	Q20
7	40	57	1.17	.32	1.00	.0	.87	-.3	.42	73.7	74.5	Q7
12	41	57	1.06	.32	1.02	.2	.94	-.1	.39	75.4	75.7	Q12
16	45	57	.60	.36	.76	-1.2	.56	-1.3	.59	84.2	81.0	Q19
14	46	57	.47	.37	.83	-.8	.71	-.7	.52	86.0	82.6	Q14
10	47	57	.33	.38	1.04	.3	.81	-.4	.40	84.2	84.1	Q10
6	48	57	.18	.40	1.01	.1	.91	.0	.40	86.0	85.8	Q6
8	49	57	.01	.42	1.05	.3	1.41	.9	.35	87.7	87.6	Q8
11	49	57	.01	.42	.93	-.2	.73	-.5	.46	87.7	87.6	Q11
5	50	57	-.18	.45	1.07	.3	.88	-.1	.37	89.5	89.3	Q5
2	54	57	-1.37	.70	.90	.0	.51	-.3	.51	96.5	96.2	Q2
9	54	57	-1.37	.70	.80	-.1	.28	-.7	.59	96.5	96.2	Q9
1	55	57	-2.00	.89	.84	.1	2.15	1.1	.41	98.2	97.7	Q1
3	55	57	-2.00	.89	.76	-.1	.29	-.3	.57	98.2	97.7	Q3
4	56	57	-3.15	1.29	2.23	1.3	.62	.3	.20	96.5	98.7	Q4
15	57	57	-4.86	2.00	MINIMUM ESTIMATED MEASURE							Q17
MEAN	46.1	57.0	-.29	.62	1.04	.2	.90	.0		85.7	86.3	
S.D.	11.3	.0	1.98	.43	.34	.8	.47	.8		11.2	9.0	

In addition, person reliability is not dependent on sample ability variance, thus, this low value may imply there is not much difference between their abilities, thus making it impossible for the samples to be discriminated into different levels. Therefore, this sample is not able to demonstrate a hierarchy of ability (Bond & Fox, 2007).

The existing variations in the person reliabilities from all schools may also be indicators of "strangeness" of construct on the part of the respondents; thus, the need to revise the test and add supplementary items or to investigate the learning environments of some of the schools reporting low person reliabilities. The question of "strangeness" can be observed to exist based on the responses to particular items in the test.

Five out of six schools measured reliability values less than 0.60. This simply indicates that there is not enough room to say for certain that the top measure can be distinguished from the bottom measure (Fisher Jr., Elbaum, & Coulter, 2010). The number of ranges in the scale can be distinguished with confidence when there the reliability value is greater. Measures with value of 0.67 will tend to vary within two groups that can be separated with 95% confidence. Values of 0.80, 0.90, 0.94, 0.96, 0.97 will vary within three, four, five, six and seven groups, respectively. Higher values will behave in the similar pattern (Fisher Jr., et al., 2010). This can be explained further by looking at the values of the person raw score to measure correlation and the item raw score to measure correlation. The person raw score to measure correlation for all schools fell within the range of 0.88 to 0.98. The item raw score to measure correlation were within -0.87 to -0.99. Both sets of values point to the fact that the proportion of very high and very low scores is low ("Winsteps Help for Rasch Analysis," 2011). Thus, there existed only a small ability difference between the respondents from these schools.

FINDINGS AND DISCUSSION

Bubble charts were drawn up for the data from all six schools. The following Figure 1 shows a bubble chart drawn for responses to the Computation Test by respondents from SMKAA.

Bubble charts are easy to read. The interesting point to observe in using the Rasch measurement model is that the difficulty of an item relative to the other items is represented by its' distance from the bottom of the scale. Items closer to the bottom are easier; those further up will be more difficult. A student will progress along the scale as far as his or her ability will carry him or her, thus this distance will identify an estimated value of a student's ability to tackle an item. As can be seen, the most difficult item was Q13, easiest was Q17, most predictable was Q20 and the least predictable was Q19. Q17 and Q13 fell in the category of *Numbers* while Q19 and Q20 fell in the category of *Measures*.

The person ability and item difficulty estimates are measured along a *logit* scale, an interval scale with consistent unit intervals between locations of items (Bond & Fox, 2007). The estimation 'measure' of difficulty level for each of these four items can be transformed in terms of percentage, for better understanding of the interpretation. Given the *logit* value for item b as α, then the percentage difficulty level can be furnished by the following procedure:

$$P(\textit{difficulty item}\, b) = \frac{e^{\alpha}}{1+e^{\alpha}}$$

Table 2 displays the measure order for all items in the test for SMKAA. The *logit* value for each item in a test can be read from the column 'measure' in the table for measure order. It can be seen that the difficulty level of items was distributed from -4.86 *logit* to 3.55 *logit* . The mean of the estimates for this set of responses from SMKAA was estimated at -0.29 *logit*.

In order to understand the difficulty students faced in tackling the item, it is important that consideration be given as to whether the errors done were conceptual or procedural errors. A conceptual error occurs when the student has insufficient understanding of "the nature of numbers or the operations involved" while a procedural error takes place due to carelessness and/or some other reasons in a situation where the student is equipped with the overall strategic understanding of actions to take (A. J. McIntosh, 2002). In particular, this paper will try to explain the difficulties students faced by looking at the errors made in the calculations. In this way, this paper will be able to furnish some elements for teachers to ponder upon in the effort to enhance the teaching techniques while at the same time help develop students develop number sense.

Most Difficult and Easiest Item

Table 3 displays the question asked in Q13 (Item 13) and Q17 (Item 15).

Table 3: Questions Asked in Q13 and Q17

Question Number	**Entry (item) Number**	**Question Asked**
17	15	If one ream of A4 paper costs RM11.35, how much would 4 reams cost?
13	13	$3\frac{3}{8}\times 3\frac{1}{6}$

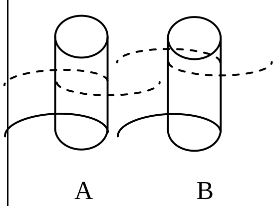
Table 4: Partially Correct Responses to Item 13

Student	Partially Correct Response	Error(s) Made	Type of Error
1	$3\frac{3}{8}\times 3\frac{1}{16}=\frac{27}{8}\times\frac{49}{16}=\frac{27\times 49}{\cancel{8}_1\times\cancel{16}_2}=\frac{1323}{2}=661\frac{1}{2}$	Students cancelled 8 and 16	Conceptual Error
2	$3\frac{3}{8}\times 3\frac{1}{16}=\frac{27}{8}\times\frac{49}{16}=\frac{27\times 49}{\cancel{8}_1\times\cancel{16}_2}=54$	Mistake in division - Students cancelled 8 and 16 in the denominator	Conceptual Error
3	$3\frac{3}{8}\times 3\frac{1}{16}=\frac{27}{8}\times\frac{49}{16}=\frac{27\times 2}{8\times 2}\times\frac{48}{16}=\frac{\cancel{34}^{17}}{\cancel{16}_8}\times\frac{\cancel{48}^{8}}{\cancel{16}_8}=\frac{136}{8}$	Mistake in multiplication Followed by mistake in division	Procedural Error
4	$3\frac{3}{8}\times 3\frac{1}{16}=3\frac{6}{16}\times 3\frac{1}{16}=9\frac{6}{16}$	Mistake in multiplication	Conceptual Error

$$\boxed{\begin{array}{r} 3986 \\ \underline{-3006} \\ 0980 \end{array}}$$

Figure 2: Sample Response for Q4

Table 5: Questions Asked in Q20 and Q6

Question Number	**Entry (item) Number**	**Question Asked**
20	17	 Containers A and B are of the same size and shape that can hold 1000 *ml* of liquid. If the liquid in the container A is half full, estimate the volume in container B.
6	6	989 + 3103 - 98

Problem Faced by Students in Tackling Q13

How difficult was Q13? Using values from Table 2, the estimated level of difficulty for item 13 can be calculated

as $P(difficulty\ \ item\ 13) = \dfrac{e^{3.55}}{1+e^{3.55}} = 0.9721$. This gives the picture that almost 97.21% of the respondents were finding this item difficult to handle. Interestingly, Item 13 was not only found to be the most difficult item for respondents from SMKAA, it was also the most difficult for the respondents from the other five schools. The *logit* value of this item for SMKAA was the smallest from among all six schools followed by SMKKD (4.27), SMKJ (4.54), SMKBBS (5.32), SMKAK (5.78) and SBPI (6.79). Therefore, it would logical to point out that the percentage of difficulty would be greater for the other schools, with the largest *logit* value contributing as the school with the highest difficulty level for Q13. The surprising thing to observe here is that the category of school does not have any effect on the performance on students in this item.

Primary school students indicated that fractions were the most complex idea they encountered during their primary school years. However, fractions must be emphasized as an important mathematical idea, because a) ability to deal effectively with fractions can greatly improve students' abilities to understand and handle real-life problems, b) the cognitive space created through learning rational numbers will provide the arena for development and expansion of the mental structures required for continued intellectual development, and c) understanding rational numbers leads to better understanding of elementary algebraic operations later on in the students' lives (M. J. Behr, Lesh, Post, & Silver, 1983). Fractions can be broken down into three categories, Proper Fractions, Improper Fractions and Mixed Numbers. As can be seen from Table 3, Q13 (item 13) was on multiplication of mixed numbers.

It was found that lower secondary school students were having problems with questions to name mixed numbers on a number line with two reference points (Noordin, Abdol Razak, & Ali, 2011; Noordin, Abdol Razak, Dollah, & Alias, 2010). In addition, lower secondary school students were also found not able to handle subtraction of mixed numbers well; they had problems with the "taking away" process in the subtraction of mixed numbers (Noordin, et al., 2011). This item 13 has now added another dimension of the mixed numbers that students may have problems with, namely application of the multiplication operation to mixed numbers. It would be an added advantage if these symptoms are researched on further by the Ministry of Education and teachers in order to inculcate better number sense of fractions in the students.

The location of an item on the bubble chart is determined by its the level of difficulty to the students. There is a need to understand whether students have developed number sense in the process. In particular, with respect to item 13, nine test papers were selected at random. From among these, only three managed to answer the item correctly. Of the remaining six, four were found to be partially correct. Table 4 displays the partially correct responses.

What can be deduced from these results? Students displayed no difficulty in converting $3\frac{3}{8}$ to $\frac{27}{8}$ and $3\frac{1}{16}$ to $\frac{49}{16}$. This meant that they had the ability to convert a mixed number into an improper fraction. With the exception of *Student 3* in Table 4, all the others had problems with the application the multiplication operation on mixed numbers. *Student 1* and *Student 2* did their division in the denominator while *Student 4* multiplied the whole number to a whole number and a fraction to a fraction. These errors were to be expected because the multiplication operator is only valid for one whole number and a fraction in the curriculum for Years 1 to 6. However, the multiplication of two mixed numbers is included in the syllabus for Form 1(*Mathematics Form 1*, 2002). That could only mean that students have not grasped the concept well enough to complete this task.

Problem Faced by Students in Tackling Q17

How difficult was the easiest item, namely Q17? Q17 (or item 15) was focused on the topic of money. Respondents were asked to calculate the cost of 4 reams of A4 paper given the price of one ream. This required the ability to multiply a decimal number up to two places with a whole number, which was the learning objective for Year 4 primary school students. Under points to note in the curriculum under this learning objective, teachers were encouraged to teach students to make sensible estimations to check answers to problem (*Mathematics Year 4*, 2006). Furthermore, under the learning objectives "use and apply knowledge of money in real life situation," the suggested learning outcomes included "multiply money to the highest product of RM10 000"(*Mathematics Year 4*, 2006).

Using the respective value from Table 2, the estimated level of difficulty for Q17 would be $P(difficulty\ \ item\ 20) = \dfrac{e^{-4.86}}{1+e^{-4.86}} = 0.0077$. This is equivalent to a percentage difficulty level of less than

1%, meaning not many respondents found this question difficult to handle and majority of the respondents were able to pass Q17 at this school. The percentage of difficulty for the other schools were greater since this item measured higher in *logit*s, namely -2.18 (SMKAK), -1.99 (SMKBBS), -1.80 (SMKJ), -1.40 (SMKD) and -1.23 (SBPI). Unlike SMKAA, only approximately 10% – 25% of the students at each of these five (5) schools found Q17 to be difficult. From among the same nine randomly selected papers, seven managed to get it perfectly correct while the other two made procedural errors in multiplication [this paper assumed these took place due to carelessness] and came up with the answers RM45.80 and RM15.40.

However, this item proved not to be the easiest item for the other schools. In fact, Q4 was favored more by majority number of schools with the percentage of difficulty ranging from approximately 3% - 11% [The measure order of Q4 ranged from a minimum value of -3.18 *logit*s to a maximum value of -2.13 *logit*s for all six schools.].

In Q4, respondents were asked to subtract 3006 from 3986, which can be done very easily without using calculations. The students did not need to do any work for this question; they could easily have used mental computation. Surprisingly, all nine (9) samples indicated work done to subtract 3006 from 3986. In fact, there were three papers that displayed the working as shown in Figure 2.

Two of the respondents left their answer as that and the other respondent concluded his answer as 980. These two respondents were fully procedural in their approach and made no judgment as to whether the answer made sense or not. According to Ghazali and Ahmad Zanzali (1999), students who understand numbers can relate well to mathematical operations and be able to use their judgment in decision making processes involving mathematic related problems.

Most Predictable and Least Predictable Item

As stated earlier on, the most predictable and the least predictable item for this SMKAA was Q20 and Q19, respectively. However, the most predictable and least predictable items were not found to be the same for all schools. Similar to SMKAA, the respondents to two other schools also found Q20 to be the most predictable item. Contrary to the choice by the respondents of SMKAA, majority of respondents found Q6 to be the least predictable. This paper will discuss both these items further. Table 5 displays the question for each of these items.

Problem Faced by Students in Tackling Q20

Q20 (item 17 in Table 5) was a problem solving question asking for estimation of volume of liquid for one container B given the height of liquid in the other given container A. The range of value within which this item was located was 1.85 to 3.28 *logit*s. This was proportional to a difficulty level of

$$P(difficulty\ item\ 17) = \frac{e^{1.85}}{1+e^{1.85}} = 0.8641 \text{ to}$$

$$P(difficulty\ item\ 17) = \frac{e^{3.28}}{1+e^{3.28}} = 0.9637\ .$$

Therefore, approximately 86.41% to 96.37% respondents were finding this item difficult to handle. What the students needed to do was just to estimate that the second container was filled up with liquid to about ¾ of the height of the container and come up with the answer that the volume of liquid in container B was $500ml + \frac{1}{2}(500ml) = (500+250)ml = 750ml$.

This simply meant that students needed to know that they had to add volume of liquid equivalent to half of the volume in container A, which was $250ml$ of liquid. Of the nine randomly selected papers, two respondents gave the answer as seen in Figure 3.

This paper is trying to figure out what would be the most the reasonable explanation for the answer. This paper can only assume that Student 1 may have used his ruler to divide the height of the container into 5 equal parts instead of six and Student 2 did not complete his work. Inability of Student 1 to estimate the height of the liquid level has impaired his ability to find the volume.

When students develop number sense, they show improvement in their understanding of numbers by being better at representing numbers, associating numbers with number systems, associating between operations of numbers, can compute more fluently and estimate reasonably (NCTM, 2000). The inability to answer Q20 indicated that students were not able to estimate size well. Inability to estimate size of numbers can impair student's ability to estimate numbers, and likewise, estimation can help to develop an understanding of number size (M. Behr & Post, 1986).

Problem Faced by Students in Tackling Q6

In Q6, students were asked to calculate the value of $989+3103-98$. Under the section "points to note" in the curriculum specifications, teachers are encouraged to remind students to do their calculation from left to right for mixed operation numerical problems involving addition and subtraction. The objective of this item was to test whether students understood that addition was the process of combining two groups of objects and subtraction was a "take away" or "difference" process between two groups of objects.

Student 1: $\boxed{\frac{4}{5}\times 1000 = 800\,ml}$ Student 2: $\boxed{\frac{1000}{2} = 500\,ml}$

Figure 3: Responses to Q20

$$\boxed{\begin{array}{rr} 989 & 4192 \\ +3103 & -\ \ 98 \\ \hline 4192 & 4094 \end{array}}$$

Figure 4: Response for Q6

More than 50% of students from four schools found this item difficult to handle. This was an interestingly surprising result.The question to ask would be “Were the students performing conceptual errors?” Of the nine randomly selected papers, two gave their answers as 3984 and 4094. Clearly, these were simply procedural mistakes. The first response was a value less 10 from the answer [3994]. To understand the reason for the difference of 100 for the second response, the following Figure 4 displays the student’s working.

This response indicated that the procedural error was done in the first section of the working by giving the answer 41 to 9+31, causing a difference of 100 to take place in the second section of the working.

CONCLUSION

How do lower secondary school students respond to simple mathematics problems? Previous studies have indicated that students find fractions difficult to handle, thus it was to no surprise that all six schools put Q13 on mixed numbers as their most difficult item. Multiplication of a fraction to a whole number is covered in Year 6 while multiplication of a fraction to a fraction (including mixed numbers) is placed in the curriculum for Form 1. The maximum ability displayed by students was to be able to convert mixed numbers to improper fractions. Majority of them were not able to piece together all previous knowledge to enable them to move forward to the next level of knowledge, that is, to find an answer to represent the product of two mixed numbers. The next two items, Q17 (easiest item for SMKAA) and Q4 (easiest item for 4 out of 6 selected schools) highlighted two points to consider, namely, a) when items are too easy, student tend to make procedural errors, and b) when students are too engrossed in the procedural work, they tend to be less appreciative of the number sense that is involved. The responses to Q20 pointed to the incapability of students to make correct judgments as to the size of the length from the floor to the upper level of liquid in the form of a fraction of a whole [part-whole concept was covered at Year 3]. The question of finding the correct answer once the fraction is determined should not be a problem because students displayed no problem with multiplying a fraction to a whole number. It was reported that a constant 20% of Grade 3 to 6 students at every level gave incorrect answers to addition and subtraction of two single-digit numbers, usually due to minor procedural errors in calculations (A. McIntosh & Dole, 2000), thus to find half of the schools not able to give correct answers to a problem involving three numbers up to three digits in Q6 would be a natural phenomena. These are minute details that have to be studied in order to improve the process of developing number sense in students at all levels.

ACKNOWLEDGEMENT

This paper would like to acknowledge the Research Management Institute (RMI) of UiTM Malaysia for funding this study and the Education Departments of both Kedah and Perlis, the principals, teachers and students of all six schools for their considerable support and time.

REFERENCES

[1] Abu Hasan, Y. (2007). Mathematics Education and Technology: Past, Present and Future – The Malaysia Experience. *The Electronic Journal of Mathematics and Technology, 1*(1), 90-94.

[2] Bana, J., & Dolma, P. (2004). *The Relationship between Estimation and Computation Abilities of Year 7 Students.* Paper presented at the 27th

annual conference of the Mathematics Education Research Group of Australasia (MERGA-27), Townsville QLD.

[3] Behr, M., & Post, T. (1986). Estimation and Children's Concept of Rational Number Size. In H. Schoen & M. Zweng (Eds.), *Estimation and Mental Computation-1986 NCTM Yearbook* (pp. 103-111). Reston, VA: National Council of Teachers of Mathematics.

[4] Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational Number Concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes* (pp. 91-125). New York: Academic Press.

[5] Bond, T. B., & Fox, C. M. (2007). *Applying the Rasch Model: Fundamental Measurement in the Human Sciences* (2nd ed.). New Jersey: Lawrence Erlbaum Associates.

[6] Fisher Jr., W. P., Elbaum, B., & Coulter, A. (2010). Reliability, Precision, and Measurement in the context of Data from Ability Tests, Surveys, and Assessments. *Journal of Physics : Conference Series 238, 012036.* doi: 10.1088/1742-6596/238/1/012036

[7] Ghazali, M., & Ahmad Zanzali, N. A. (1999). *Assessment of School Childrens' Number Sense*. Paper presented at the International Conference on Mathematics Education into the 21 Century: Societal Challenges: Issues and Approaches, Cairo, Egypt.

[8] Ghazali, M., Idros, S. N., & McIntosh, A. (2004). From Doing to Understanding: An Assessment of Malaysian Primary Pupils' Number Sense with Respect to Multiplication and Division. *Journal of Science and Mathematics Education in South East Asia, 27*(2), 92-111.

[9] . *Mathematics Form 1*. (2002). Ministry of Education Malaysia.

[10] . *Mathematics Year 1*. (2002). Ministry of Education Malaysia.

[11] . *Mathematics Year 2*. (2003). Ministry of Education Malaysia.

[12] . *Mathematics Year 3*. (2003). Ministry of Education Malaysia.

[13] . *Mathematics Year 4*. (2006). Ministry of Education Malaysia.

[14] . *Mathematics Year 5*. (2006). Ministry of Education Malaysia.

[15] . *Mathematics Year 6*. (2006). Ministry of Education Malaysia.

[16] McIntosh, A., & Dole, S. (2000, October 15 - 17). *Number sense and mental computation: Implications for numeracy.* Paper presented at the ACER Research Conference 2000, Brisbane, Australia.

[17] McIntosh, A. J. (2002, July 7 - 10). *Common Errors in Mental Computation of Students in Grades 3-10.* Paper presented at the Mathematics Education in the South Pacific, Auckland.

[18] NCTM. (2000). Principles and Standards for School Mathematics. In J. Bana & P. Dolma (Eds.), *The Relationship between Estimation and Computation Abilities of Year 7 Students*. Reston, VA: P. Merga27 Conference Proceedings

[19] Noordin, N., Abdol Razak, F., & Ali, N. (2011, June 22-23 2011). *Handling Mixed Numbers on a Number Line - A Problem among Lower Secondary School Students.* Paper presented at the ASEAN Conference on Scientific and Social Science Research (ACSSSR2011), Penang, Malaysia.

[20] Noordin, N., Abdol Razak, F., Dollah, R., & Alias, R. (2010, July 25 2010). *Identifying Equivalent Pictorial Representation of a Fraction and a Whole – a Problem for 13-Year Olds in Malaysia.* Paper presented at the ASEAN Conference on Environment-Behaviour Studies (AcE-Bs 2010), Sarawak, Malaysia.

[21] Reys, R. E. (1984). Mental Computation and Estimation: Past, Present, and Future. *The Elementary School Journal, 84*(5), 546-557.

[22] Segovia, I., & Castro, E. (2009). Computational and Measurement Estimation: Curriculum Foundations and Research carried out at the University of Granada, Mathematics Didactics Department. *Electronic Journal of Research in Educational Psychology, No 17, 7*(1), 499-536.

[23] . Sejarah Sekolah Berasrama Penuh. (n.d.) Retrieved July 30 2011, from http://bpsbpsk.webs.com/sekolahberasramapenuh.htm

[24] Tall, D. (2008). The Transition to Formal Thinking in Mathematics. *Mathematics Education Research Journal, 20*(2), 5-24.

[25] Tsao, Y. (2004). Exploring the Connections among Number Sense, Mental Computation Performance, and the Written Computation Performance of Elementary Preservice School Teachers. *Journal of College Teaching and Learning, 1*(12).

[26] Winsteps Help for Rasch Analysis. (2011). Retrieved from 99.236.93.8/winman/index.htm?disfile.htm