IMF Recommended Debt Sustainability Threshold for Nigeria. Is it Growth Augmenting? An Optimization Algorithm Approach

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Abstract: Sustainable debt is the level of debt which allows a debtor country to meet its current and future debt services obligations in full, without recourse to further debt relief or rescheduling, avoiding accumulation of arrears, while allowing an acceptable level of economic growth or without compromising growth. IMF debt sustainability threshold is seen as a measure of a countries solvency, that is, a countries ability to ensure that accumulated debts are serviced without defaulting and if countries are able to sustain their debt servicing overtime Generally, the study examines the country's borrowing space under the International Monetary Fund (IMF) debt sustainability threshold of 45% for Nigeria and other low-middle income countries. To achieve the stated objectives, two variant of optimization algorithms namely The Gauss, Berndt, Hall, Hall and Hausman, and Marquardt algorithms, respectively were used as estimators. The Gauss, Berndt, Hall, Hall and Hausman commonly referred to as the Gauss-Newton/BHHH is specifically designed for maximum likelihood problems, while Marquardt algorithm modifies the Gauss-Newton algorithm by adding a correction matrix to the Hessian approximation. Two variant models namely the Least Square (LS) and the Maximum Likelihood-Autoregressive Conditional Hetersocesdasticity (ML-ARCH) were used to cross-match the Generalized Linear Model (GLM) which is the primary model for the study. The data used covers a period of 48 years (1965-2013). It was collected from the online database of the Central Bank of Nigeria (CBN); National Accounts archives of National Bureau of Statistics, and human resources data of Food and Agricultural Organization statistics. From the findings, the study showed that IMF recommended sustainability threshold is not growth augmenting but will act as a hindrance to economic growth in Nigeria. The debt sustainability threshold of 45% for Nigeria is not growth supportive. Our estimate showed that maintaining such threshold drives output to a negative growth of (-19.5%) and (-27.9%), respectively. In all, the study established that sustainability is definable within the country's growth objective in contrast with debt sustainability analysis which is traditionally based on solvency. In this contest, the study generated an alternative definition of debt sustainability as that threshold which generates maximum debt-output ratio and ensures a steady state and maximum attainable growth in an economy, that is, threshold that ensures a zero growth deficit. It was recommended that sustainability threshold of debt acquisition should not be defined in a vacuum. It should be sacrosanct with the country's sustainable economic growth objective. That is debt acquisition that is not only serviceable, but ensures a maximum attainable growth.

Keywords: External Debt Sustainability Threshold, Growth Augmenting, IMF, Nigeria, Optimization Approach.

Introduction

The IMF debt sustainability threshold is seen as a measure of a countries solvency, that is, a countries ability to ensure that accumulated debts are serviced without defaulting and if countries are able to sustain their debt servicing overtime (Akyüz 2007). It therefore implies that the economy is well-off and debt contracted is not detrimental to growth. The country can therefore contract more loans. Different debt sustainability threshold for low income countries, lower middle income countries, middle income countries and high income countries has been specified by IMF. Developing countries who fall within the categories of low income, lower middle income and middle income countries have tried to maintain this threshold or even contract below this threshold to ensure that they do not default in debt servicing. But in recent times, it has been observed that these countries are experiencing worse economic crisis; low growth rate, large number of citizens living in poverty, high unemployment rate, and incessant strike action by labour as government cannot meet with some of their demands. A large number of these countries growth objectives are far from being achieved.

Nwankwo (2014) noted that in Nigeria and other developing countries, fiscal deficits are mainly driven by the need for huge capital requirement, which are not usually available through current receipts, to finance critical infrastructure projects necessary to drive and position the country for growth and development. He went further to state that the goal of fiscal deficits and by implication, public debt in Nigeria is the pursuit of rapid economic growth towards improving the living standard of the populace. This implies that the goal of public debt (External debt or/and Domestic debt) in Nigeria is to ensure achievement of its growth objectives and improve the living standard of its populace through infrastructure projects. Given this background, the question now is; has Nigeria witness or achieve any tremendous growth rate, given the fact that in recent times, Nigeria has tried to maintain a low debt sustainability threshold that would ensure non-default in servicing its debt obligation, and help position it for growth and development?

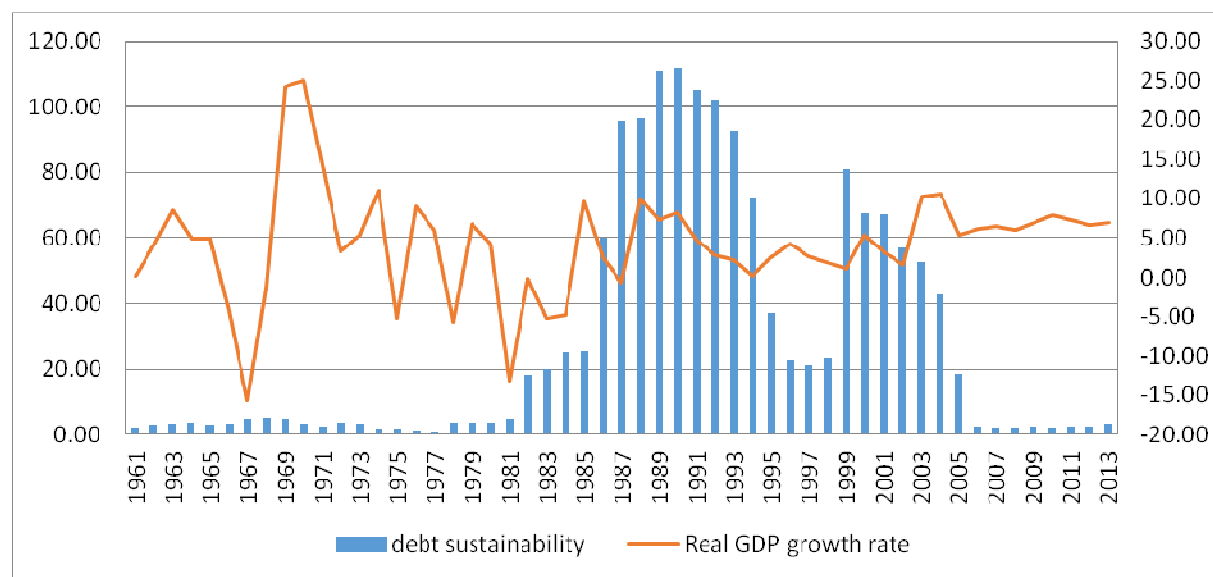Below is chart showing the relationship between Debt Sustainability and Economic Growth in Nigeria.

Figure 1: Nigeria's Debt Sustainability Threshold and Economic Growth Rate
Source: Author using data from CBN and, DMO

From figure 1 above, it can be observed that in the early years of foreign debt contraction (1963/1964 to 1981/1985) debt sustainability threshold was very low, but most of these years recorded a negative growth rate even as low as -15.7% in 1967 and -13% in 1981, with debt sustainability threshold of 4.8% and 4.9% respectively. With debt relief in 2006, one would have expected the growth rate to rise since the stock of debt left to service has reduced and debt serving which is seen to retard growth or be detrimental to growth has also reduce, rather low growth rate was recorded.

Since debt sustainability and economic growth are not known to have a pattern of relationship given the Nigerian experience, it therefore implies that the IMF use of debt sustainability as a measure of ensuring that debt is not detrimental to growth is questionable. The question now is, the IMF debt sustainability threshold benchmark of 40% 45% for low and low-middle income countries, respectively growth-supportive as in the case of Nigeria?

This study therefore tries to ascertain if the debt sustainability threshold of 40% and 45% benchmarked by IMF for Nigeria is growth augmenting. This will be done by employing the Gauss-Newton Berndt, Hall, Hall, Hausman optimization algorithm alongside Marquardt algorithm in determining the critical levels of external debt sustainability threshold that is consistent with Nigeria's growth objective. This will act as a step to re-defining Nigeria's debt sustainability threshold that will be consistent with growth objectives.

Conceptual Issues of External Debt Sustainability

Sustainable debt is the level of debt which allows a debtor country to meet its current and future debt services obligations in full, without recourse to further debt relief or rescheduling, avoiding accumulation of arrears, while allowing an acceptable level of economic growth or without compromising growth. Akyüz (2007) defined external debt sustainability as the ability of a country to meet the current and future external obligations of both private and public sectors without running into arrears, recourse to debt-rescheduling and eventually a drastic balance-of-payment adjustment. Debt is sustainable if government generates an adequate level of primary surplus when the actual debt ratio exceeds the target ratio (Akyüz 2007; Croce and Juan-Ramón 2003; IMF 2003)

External Debt Sustainability in Nigeria

After the debt relief in 2006, Nigeria government aimed at ensuring that the country does not relapse into debt unsustainability. This initiative led to the commencement of the conduct of annual National Debt Sustainability Analysis (DSA), using the debt sustainability framework developed by the IMF. The purpose of this exercise is to assess or analyse the Nigeria's ability/capacity to meet its current and future debt obligation, its policy goals, and service the ensuing debt without unduly large adjustments, which could otherwise compromise growth and development (DMO 2013, Nwankwo 2014). The government adopted the Fiscal Responsibility Act of 3%, that is, accumulation of external debt at 3% debt sustainability threshold. Overtime, Nigeria has serviced its debt without defaulting, thereby making IMF to rank the country as a low-middle income country, which implies that the country can borrow from external source above 40% debt sustainability threshold.

External debt sustainability analysis is generally conducted in the context of medium-term scenarios. These scenarios are numerical evaluations that take account of the expectations of the behavior of economic variables and other factors to determine the conditions under which debt and other indicators would stabilize at reasonable levels, the major risks to the economy and the need and scope for policy adjustment. There are various indicators for determining a sustainable level of external debt. These indicators measures the country's 'solvency' in that they consider the stock of debt at certain time in relation to the country's ability to generate resources to repay the outstanding balance. They include; Debt/GDP ratio, Debt/Export ratio and Debt/Government ratio. Another set of indicators focuses of the short term liquidity requirements of the country with respect to its debt service obligation. Below is a table showing the various indicators.

Table 1: External Debt Indicators

Medium-Term Indicator 'Solvency'	Sustainability Threshold (%)	Short-Term Indicators (Liquidity)	Sustainability Threshold(%)
Debt to GDP ratio	40 and 45	Foreign Debt Service to Export ratio	20
Foreign Debt to Exports ratio	150	Government Debt Service to Current Fiscal Revenue ratio	20
Government Debt to Current Fiscal Revenue ratio	250		

Source: Debt Management Office, 2013 and 2014

From table 1, the indicator, debt-GDP ratio of 40% is the debt sustainability threshold recommended by IMF for Nigeria. This is one of the main indicators used for assessing how solvent (solvency) the country is and their ability to service their debt without default. 150% sustainability was recommended for foreign debt to export ratio while 250% recommendation was made for Government Debt to fiscal revenue ratio.

Theoretical framework

This study adopted the classical theory of optimization which centered on the traditional calculus approach involving the minima and maxima. The application of traditional calculus to optimization problem anchors on weiestrass's theorem, which states that every function which is continuous within a closed range R of variables (x_1, x_2, ……x_n) possesses a largest and smallest value within the domain of the function (Ekanem and Iyoha, 2000). The calculus approach however, is restricted to functions that are continuous and differentiable. One of the great advantage of the calculus approach is that it allows us to obtain not only the optimal condition in terms of minimum or maximum of the mathematical function, but also the value of the independent variables that cause the function to be optimized (Ekanem and Iyoha, 2000)

One of the principal applications of derivatives is in the determination of relative extrema in a function. For the purpose of this study, constrained exterma was used since an optimization problem becomes useful when budget constraint is introduced to accompany the objective function to be maximized. Lancaster (1969) noted that 'optimizations subject to constraints has been considered by many as defining the essential nature of economics'. The constraint establishes a relationship among the x_i decision variables. It is designed to invoke certain limiting factors inherent in the optimization problem. Given a function of one variable $y = f(x)$, the first order condition for the existence of an extremum is that $f''(x) = 0$. The second order condition is that $f''(x) < 0$ to establish sufficient condition for a maximum or $f''(x) > 0$ for a minimum.

Methodological Framework
In order to ascertain if the debt sustainability threshold is growth augmenting, the study used a quadratic hill-climbing technique of external debt. The hill-climbing technique is particularly useful when models may be estimated using second derivative method – of which the debt sustainability threshold evaluation requires. The critical levels of external debt here mean an amount of external debt accumulation that potentially, becomes either a hindrance or economic growth-augmenting.

Conceptually, the definition of debt sustainability threshold in the current study is predicated on whether debt acquisition is growth-augmenting or binding-constraint. Thus, a debt sustainability threshold in this context will occur at that point where debt acquisition stop being growth-augmenting. This differs significantly, from the use of DSA (debt sustainability analysis) by the International Monetary Fund (IMF) which merely uses debt-GDP ratio as measure of countries' solvency or the ability of the economy to repay every one unit of debt obligations – of which the central focus is a guide against default irrespectively of whether the obligation supports growth or not.

In essence, our approach is not only interested in sustainable debt (solvency) or impact analysis (how), but also, and more importantly too, interested in sustainable development – debt for sustainable growth, a tool for understanding debt-motivated economic crises.

The model
Let (g) be growth rate of real gross domestic product (GDP); (dx) is external debt stock-GDP ratio; while (λ) is a constant. To model the growth-augmenting; growth-binding constraint; and growth-stabilizing impetus of Nigeria's external debt stock, we evaluate the critical level of Nigeria's external debt by estimating the quadratic function in case (1) below. Where (α_0 and β_0) are coefficients. The general form of the model is given as:

$$f(dx) = \beta_0 dx^2 + \alpha_0 dx + \lambda_0 \tag{1}$$

From case (1), the critical value of debt acquisition for Nigeria (dx) occurs at the point where:

$$dx = f\left(-\frac{\alpha_0}{2\beta_0}\right) \tag{2}$$

The primary objective of estimating equation (2) is to determine if Nigeria's external debt obligation has a critical point and at what point it occurs. That is, to establish if the country's debt obligation has a minimum or a maximum level since either of this outcomes has varying implications for the economy. For example, if external debt stock has a maximum, it means that growth in debt stock will continue to augment growth up to a point where it becomes a constraint, while the minimum level will mean that debt at the early stage it retards growth up to the point where it infinitely augments growth. Case (2) therefore is an important element of this study because it justifies our argument that non-guided acquisition of debt is as detrimental to the economy as non-acquisition of debt – our assertion that debt is a necessary evil in every economy, but needs to be optimized.

Growth-maximising external debt

For external debt to maximise economic growth, the necessary condition for the external debt function in case (1) requires that the linear growth-augmenting external debt elasticity/parameter (β_0) be less than zero as in case (3) below.

$$if\ \beta_0 < 0, then\ the\ maximum\ limit\ of\ (external\ debt)\ dx = f\left(-\frac{\alpha_0}{2\beta_0}\right) \qquad (3)$$

If the coefficient of the non-linear external debt acquisition (β_0) is negative, then external debt accumulation has a maximum within which it is growth-augmenting. Econometrically, this is obtained by taking the marginal change in growth rate of domestic output (g) resulting from one unit change in the accumulation of external debt stock (dx) as shown in case (4). The condition for growth-augmenting external debt accumulation is that a unit increase/decrease in external debt brings a proportionate increase/decrease in domestic output, with the limit being the point where the diminishing return to debt accumulation sets in, ceteris paribus.

$$\frac{\partial g}{\partial (dx^2)} < 0 \qquad (4)$$

Debt as growth-binding constraint

For external debt to retard economic growth, the function in case (1) requires that the external debt elasticity/parameter (α_0) be less than zero as in case (5).

$$if\ \alpha_0 < 0, then\ the\ minimum\ limit\ of\ external\ debt\ (dx)\ is = f\left(-\frac{\alpha_0}{2\beta_0}\right) \qquad (5)$$

Again, the condition for debt-growth-binding constraint is that a unit increase/decrease in external debt brings a proportionate decrease/increase in domestic output, with the limit factors being the point where the diminishing return to debt accumulation sets in, ceteris paribus as shown in case (6).

$$\frac{\partial g}{\partial (dx^2)} < 0 \qquad (6)$$

Debt at Optimal Level

Optimal level of debt is both the necessary and sufficient (second-order condition) condition for debt to either be augmenting or constraining economic growth. At that point the marginal return on debt to the domestic economy is zero. The economy maintains a stable equilibrium such that any addition or subtraction to debt stock leaves the economy worse-off. Addition to external debt accumulation leads to negative growth and in the extreme leads to debt-induced economic repression; equivalently any reduction in debt will lead to economic growth-backtracking – positive growth. This is estimated as the second-order derivative of the function in case (1) as shown in case (7).

$$\frac{\partial^2 (g)}{\partial (dx^2)} = 0 \qquad (7)$$

where (∂^2) is the second-order partial derivative of real economic growth (g) with respect to (w.r.t) external debt stock (dx^2), while (∂) is the partial derivative of external debt stock (dx)

To determine the optimal debt (dx) level of Nigeria's external debt that is consistent with the country's objective, we evaluated the second-order (∂^2) approximation of the quadratic function in case (8.1). This ensures that the parameters are stable over time as to deliver a warranted stable equilibrium condition in economic growth.

$$\partial(lng, 2) = [\alpha_1(\partial ln(dx)) + \beta_1(\partial ln(dx^2))] + \lambda_1 + \mu_1 \qquad (8)$$

To determine the growth level that is optimal and consistent with steady growth, the equation in case (8.3) was evaluated at the point where:

$$\frac{\partial^2(g)}{\partial(dx)} = 0 \qquad (8.1)$$

Estimation technique

The estimation used two variant of optimization algorithms namely Gauss-Newton/BHHH, and Marquardt algorithms, respectively. The Gauss, Berndt, Hall, Hall and Hausman commonly referred to as the Gauss-Newton/BHHH is specifically designed for maximum likelihood problems, while Marquardt algorithm modifies the Gauss-Newton algorithm by adding a correction matrix (or ridge factor) to the Hessian approximation. The ridge correction handles numerical problems when the outer product is near singular and may improve the convergence rate.

These were estimated using the Generalized Linear Model (GLM) in E-views 8.1 platform. The general form of the model is shown in equation (8.2). In addition, we cross-matched the GLM with Least square and ML-ARCH estimators to see if we can trace any significant difference between them and our benchmark estimator (GLM).

$$\partial \ln(g) = \Big[\alpha_0 \big(\partial \ln(dx) \big) + \beta_0 \big(\partial \ln(dx^2) \big) \Big] \pm \lambda_0 + \mu_0 \qquad (8.2)$$

The Data

The data used covers a period of 48 years (1965-2013). It was collected from the online database of the Central Bank of Nigeria (CBN); National Accounts archives National Bureau of Statistics, 1965-2000; and human resources data of Food and Agricultural Organization statistics.

Presentation of Research findings

Table 2: Summary of growth-augmenting and growth-constraint of external debt accumulation

Eq Name:	EQ01	EQ02	EQ03
Method:	**GLM**	**LS**	**ML-ARCH**
Dep. Var:	$\Delta(LOG(G))$	$\Delta(LOG(G))$	$\Delta(LOG(G))$
C	0.049703	0.049703	0.080151
	(0.0456)	(0.0456)	(0.0335)*
	[1.0910]	[1.0910]	[2.3904]*
$\Delta(LOG(DX))$	0.869555	0.869555	1.428495
	(0.3292)**	(0.3292)*	(0.1871)**
	[2.6415]**	[2.6415]*	[7.6348]**
$\Delta(LOG(DX)^2)$	-0.029987	-0.029987	-0.050772
	(0.0136)*	(0.0136)*	(0.0073)**
	[-2.2079]*	[-2.2079]*	[-6.9373]**
R-squared:	NA	0.1482	0.0463
F-statistic:	NA	4.2632	NA

Discussion of findings

The objective is evaluated and analysed from table 2. For proper conceptualisation of growth-augmenting threshold, we evaluated equations (2) and (4) and obtain the minimum or maximum value of external debt required for sustainable growth by substituting the parameters of linear and non-linear debt, respectively from table 2. Growth-augmenting external debt threshold is evaluated as:

$$-\frac{a_0}{(2\beta_0)} = -\frac{0.87}{(2*(-0.03))} = 14.5 \tag{10}$$

The attainable growth associated with the 14.5% external debt composition of the country is obtained by substituting the value in case 10 into case 1 as shown below.

$$f(14.5) = -0.03.(14.5)^2 + 0.87.(14.5) + 0 = 6.3\% \tag{11}$$

Combining the result in table 4.0.1 with the estimated growth-augmenting threshold in case (10) shows that at the early stage of debt acquisition, its contribution to output growth is rising, but at a decreasing rate, hence the alternating signs between the linear (positive) and non-linear (negative) debt coefficients. That is, as time progresses a negative non-linear impact of debt stock on growth is experienced showing that there is a poor application of loan such that the earlier benefit derived is gradually eroded. This is consistent with the findings by (Ayadi & Ayadi 2008) that after the initial condition that led to loan acquisition the subsequent indiscriminate borrowings and its cumulative impact takes effect when debt stock significantly depressed output growth.

Furthermore, case (11) shows that the maximum attainable growth rate when external debt composition of the economy approaches a maximum limit of 14.5% is 6.3%. Beyond this point the aforementioned depressed output growth rate is set in motion by any addition or reduction in external debt.

Debt becomes a binding constraint when debt acquisition pushes growth to exceed its optimal level or when growth backtracking shown in cases (13) and (14). Case (13) shows that an increase in debt-GDP ratio beyond the optimal (29.0%) pushes the economy to a minimum negative growth of about -0.4%, amounting to debt-growth contagion.

$$f(29.5) = -0.03.(29.5)^2 + 0.87.(29.5) + 0 = -0.4 \tag{13}$$
$$f(13.0) = -0.03.(13.0)^2 + 0.87.(13.0) + 0 = 6.2 \tag{14}$$

The implication of the foregoing is that external debt accumulation has maximum limit that is consistent with Nigeria's economic growth which is estimated to be in the neighborhood of about 14.5% of the gross domestic output. It is also worthy of note that the estimated maximum threshold converges around 14% to 15% irrespective of the chosen estimation techniques as shown in table 2.

The conclusion reached under growth-augmenting and growth-constraint debt accumulation is nonetheless a preliminary conclusion. Therefore there is need for a second order or sufficient condition to be able to show that debt has reached its maximum limit as to have a deleterious effect on growth. This is shown in case (15).

$$f(29) = -0.03.(29)^2 + 0.87.(29) + 0 = 0 \tag{15}$$

At the optimal level as shown in case (15), the parameter/coefficient of the non-linear debt acquisition statistically (see table 2) equals zero. To evaluate this at equations (8) and (8.1) reveals that when the growth rate of external debt stock approaches 29% the economy reaches zero growth rate.

Evaluation of Nigeria's debt sustainability

Two important questions are addressed under this sub-section: the sustainability of Nigeria's current debt and the IMF benchmarked debt sustainability threshold for Nigeria and other low and lower-middle income countries.

The current Nigeria's external debt position which stands at about 3% of the total GDP has an attainable real growth rate of 2.3% which that it is growth-augmenting but leaving a growth deficit of (-4.0%) – the gap between the actualized and the optimal growth rate (see Table 2). However the country is under borrowing, that is, the current external debt position is not Pareto optimal given that a gradual increase in the country's external borrowing space is capable of generating higher output growth.

On the other hand, the IMF recommended debt sustainability threshold of about 40% and 45% for Nigeria and other low and low-middle[1] income countries, respectively, are not growth supportive. Our estimate shows that maintaining such threshold drives output to a negative growth of (-19.5%) and (-27.9%), respectively (See table 3).

Table 3: Critical levels of external debt (Sustainability Threshold) and the Maximum attainable output growth rate

External debt-GDP ratio %	Attainable Real economic growth rate	Growth gap	External debt-GDP ratio %	Attainable Real economic growth rate	Growth gap	External debt-GDP ratio %	Attainable Real economic growth rate	Growth gap
0.5	0.4	-5.9	15.5*	6.3	0.0	30.5	-1.4	-7.7
1.0	0.8	-5.5	16.0	6.2	-0.1	31.0	-1.9	-8.2
1.5	1.2	-5.1	16.5	6.2	-0.1	31.5	-2.4	-8.7
2.0	1.6	-4.7	17.0	6.1	-0.2	32.0	-2.9	-9.2
2.5	2.0	-4.3	17.5	6.0	-0.3	32.5	-3.4	-9.7
3.0	2.3	-4.0	18.0	5.9	-0.4	33.0	-4.0	-10.3
3.5	2.7	-3.6	18.5	5.8	-0.5	33.5	-4.5	-10.8
4.0	3.0	-3.3	19.0	5.7	-0.6	34.0	-5.1	-11.4
4.5	3.3	-3.0	19.5	5.6	-0.7	34.5	-5.7	-12.0
5.0	3.6	-2.7	20.0	5.4	-0.9	35.0	-6.3	-12.6
5.5	3.9	-2.4	20.5	5.2	-1.1	35.5	-6.9	-13.2
6.0	4.1	-2.2	21.0	5.0	-1.3	36.0	-7.6	-13.9
6.5	4.4	-1.9	21.5	4.8	-1.5	36.5	-8.2	-14.5
7.0	4.6	-1.7	22.0	4.6	-1.7	37.0	-8.9	-15.2
7.5	4.8	-1.5	22.5	4.4	-1.9	37.5	-9.6	-15.9
8.0	5.0	-1.3	23.0	4.1	-2.2	38.0	-10.3	-16.6
8.5	5.2	-1.1	23.5	3.9	-2.4	38.5	-11.0	-17.3
9.0	5.4	-0.9	24.0	3.6	-2.7	39.0	-11.7	-18.0
9.5	5.6	-0.7	24.5	3.3	-3.0	39.5	-12.4	-18.7
10.0	5.7	-0.6	25.0	3.0	-3.3	40.0	-13.2	-19.5
10.5	5.8	-0.5	25.5	2.7	-3.6	40.5	-14.0	-20.3
11.0	5.9	-0.4	26.0	2.3	-4.0	41.0	-14.8	-21.1
11.5	6.0	-0.3	26.5	2.0	-4.3	41.5	-15.6	-21.9
12.0	6.1	-0.2	27.0	1.6	-4.7	42.0	-16.4	-22.7
12.5	6.2	-0.1	27.5	1.2	-5.1	42.5	-17.2	-23.5
13.0	6.2	-0.1	28.0	0.8	-5.5	43.0	-18.1	-24.4
13.5*	6.3	0.0	28.5	0.4	-5.9	43.5	-18.9	-25.2
14.0*	6.3	0.0	29.0	0.0	-6.3	44.0	-19.8	-26.1
14.5*	6.3	0.0	29.5	-0.4	-6.7	44.5	-20.7	-27.0
15.0*	6.3	0.0	30.0	-0.9	-7.2	45.0	-21.6	-27.9

/*/ shows the critical levels of external debt that generates steady state growth rate of 6.3%

[1] Nigeria formally exited the low income to low-middle status in 2013. The implication of that is that the country now have a larger borrowing space or debt sustainability threshold of about 40% from 45% for the low income countries.
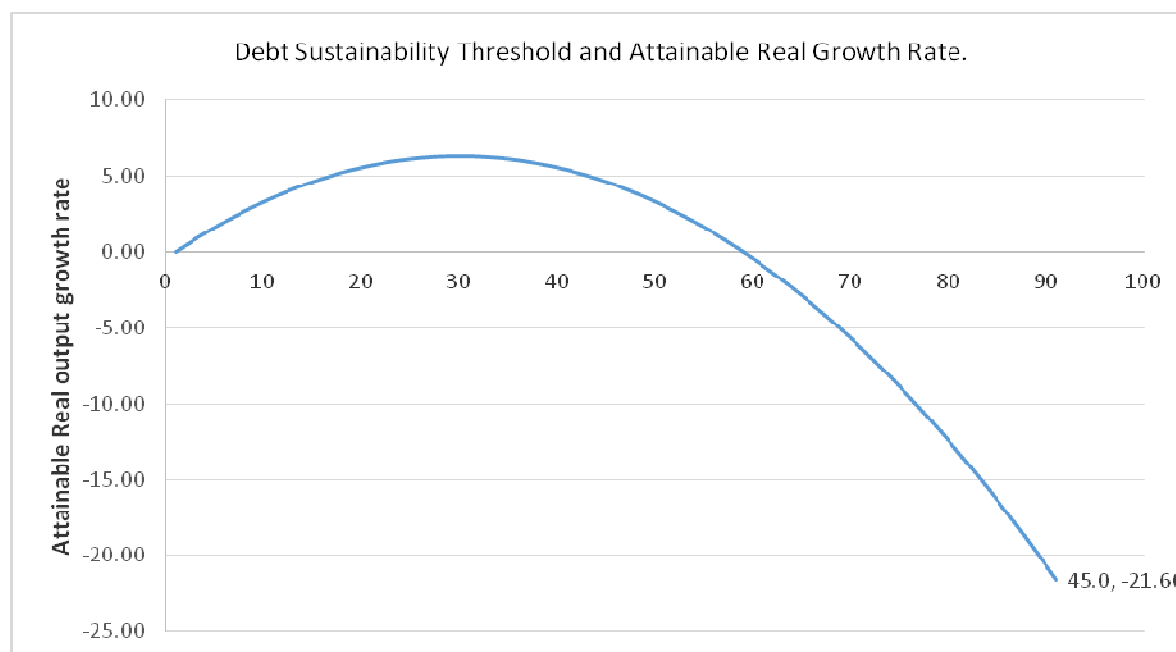
Figure 2: Real output growth rate and external debt sustainability threshold
Source: Author's simulation based on the econometric estimation of Nigeria's optimal and critical debt

4.4 Re-defining Nigeria's debt sustainability threshold

The Nigeria's borrowing space is strictly defined domestically and internationally by the fiscal responsibility act of 2007 and the International Monetary Fund debt sustainability threshold of 45% for low-middle income countries, respectively. The threshold for under the fiscal responsibility act is 3%, while the IMF threshold is 45%.

Table 2 and figure 1 showed that the IMF debt sustainable threshold of 45% for Nigeria has an attainable real growth rate of -26.1%, therefore not growth augmenting but acts as a binding constraint to growth, and which also leaves the economy with a growth gap of (-27.9%).

Given this technical flaws, we re-defined a sustainability threshold in relation to sustainable development. Thus, debt sustainability threshold is the maximum debt-output ratio that ensures a steady state and maximum attainable growth in an economy which also ensures a zero growth deficit. In the case of Nigeria, the sustainability threshold occurs at the 14.5% which corresponds with the highest and optimal growth rate of 6.3%. Summarily a sustainable debt is achieved when the growth deficit (the difference between highest attainable growth rate and actual growth rate) in an economy is zero.

Summary and Conclusion

Generally, the study examines the country's borrowing space under the International Monetary Fund (IMF) debt sustainability threshold of 45% for Nigeria and other low-middle income countries. To achieve the stated objectives, two variant of optimization algorithms namely The Gauss, Berndt, Hall, Hall and Hausman, and Marquardt algorithms, respectively were used as estimators. The Gauss, Berndt, Hall, Hall and Hausman commonly referred to as the Gauss-Newton/BHHH is specifically designed for maximum likelihood problems, while Marquardt algorithm modifies the Gauss-Newton algorithm by adding a correction matrix to the Hessian approximation. Two variant models namely the Least Square (LS) and the Maximum Likelihood-Autoregressive Conditional Hetersocesdasticity (ML-ARCH) were used to cross-match the Generalized Linear Model (GLM) which is the primary model for the study.

From the findings, the study showed that the IMF recommended debt sustainability threshold of about 40% and 45% for Nigeria is not growth supportive. Our estimate showed that maintaining such threshold drives output to a negative growth of (-19.5%) and (-27.9%), respectively

In all, the study established that sustainability is definable within the country's growth objective in contrast with debt sustainability analysis which is traditionally based on solvency. In this contest, the study generated an alternative definition of debt sustainability as that threshold which generates maximum debt-output ratio and ensures a steady state and maximum attainable growth in an economy, that is, threshold that ensures a zero growth deficit.

Therefore the IMF sustainability threshold of 45% is a binding constraint to economic growth as well as non-optimal.

Recommendation

Based on the research findings above, the study recommends that
1. Sustainability threshold of debt acquisition should not be defined in a vacuum. It should be sacrosanct with the country's sustainable economic growth objective. That is debt acquisition that is not only serviceable, but ensures a maximum attainable growth.

2. Countries should ensure that borrowed funds are productive in other to service their debts without defaulting and achieve their growth objectives.

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